

Task and Motion Planning

A Brief Tutorial

ICAPS 2026 Summer School

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Goals for Today

Convey **key intuition!**

What is TAMP?

When and why is it hard?

What are main ideas behind common approaches?

Not an exhaustive survey

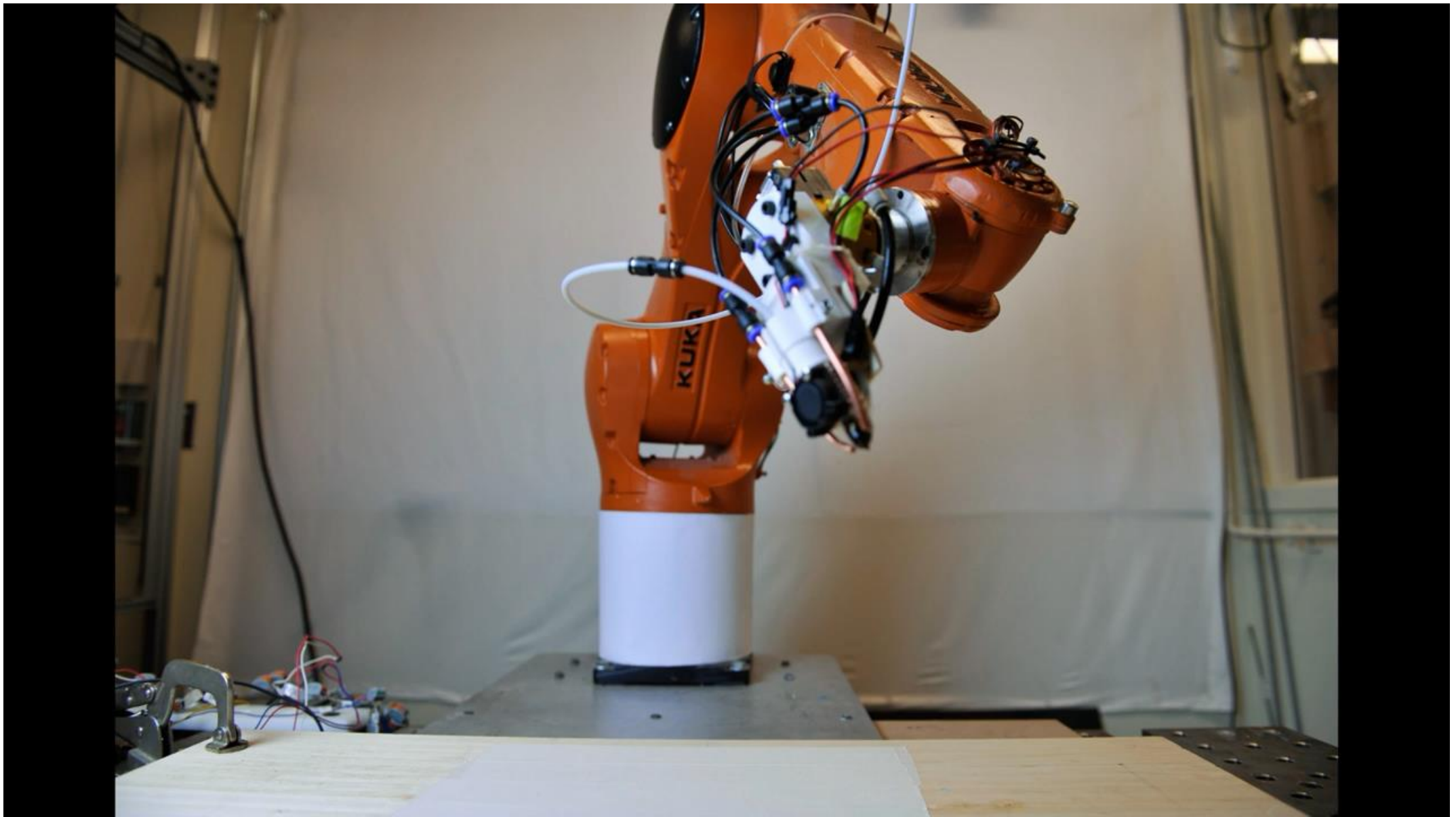
But If Surveys Are Your Thing...

“Integrated Task and Motion Planning.” Caelan Reed Garrett, Rohan Chitnis, Rachel Holladay, Beomjoon Kim, Tom Silver, Leslie Pack Kaelbling, Tomás Lozano-Pérez. ARCRAS 2021.

“A Survey of Optimization-based Task and Motion Planning: From Classical To Learning Approaches.” Zhigen Zhao, Shuo Cheng, Yan Ding, Ziyi Zhou, Shiqi Zhang, Danfei Xu, Ye Zhao. IEEE/ASME Transactions on Mechatronics 2024.



Combined task and motion planning through an extensible planner-independent interface layer. Siddharth Srivastava, Eugene Fang, Lorenzo Riano, Rohan Chitnis, Stuart Russell, Pieter Abbeel. ICRA 2014.



Scalable and Probabilistically Complete Planning for Robotic Spatial Extrusion. Caelan Garrett, Yijiang Huang, Tomás Lozano-Pérez, Caitlin Mueller. RSS 2020.

Morning Agenda

- From Task Planning to Bilevel Planning
- Introduction to Motion Planning
- Multi-Modal Motion Planning and Beyond
- TAMP in the Real World (TipTop)

Recall: Task Planning



Blocks World

Plan length: 28

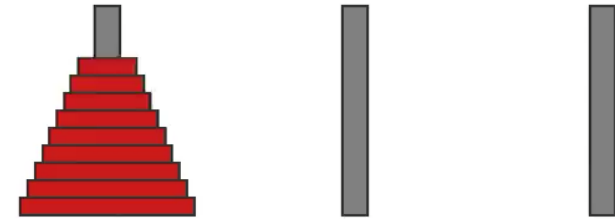
Planning time: 0.12 s



Sokoban

Plan length: 167

Planning time: 0.25 s



Hanoi

Plan length: 579

Planning time: 0.22 s

Planning with Fast Downward (<https://www.fast-downward.org>)
Rendering and simulation with PDDLgym (<https://github.com/tomsilver/pddlgy>)

Recall: Task Planning

Operator-PickFromTable:

Parameters: [**?b** - block,
 ?r - robot]

Preconditions: {GripperOpen(**?r**),
 OnTable(**?b**)}

Add effects: {Holding(**?b**)}

Delete effects: {GripperOpen(**?r**),
 OnTable(**?b**)}

Recall: Operator Grounding

Operator-PickFromTable:

Parameters: [$?b$ - block,
 $?r$ - robot]

Preconditions: {GripperOpen($?r$),
OnTable($?b$)}

Add effects: {Holding($?b$)}

Delete effects: {GripperOpen($?r$),
OnTable($?b$)}

$?b \rightarrow$ b1

$?r \rightarrow$ 

Substitution

Operator-PickFromTable:

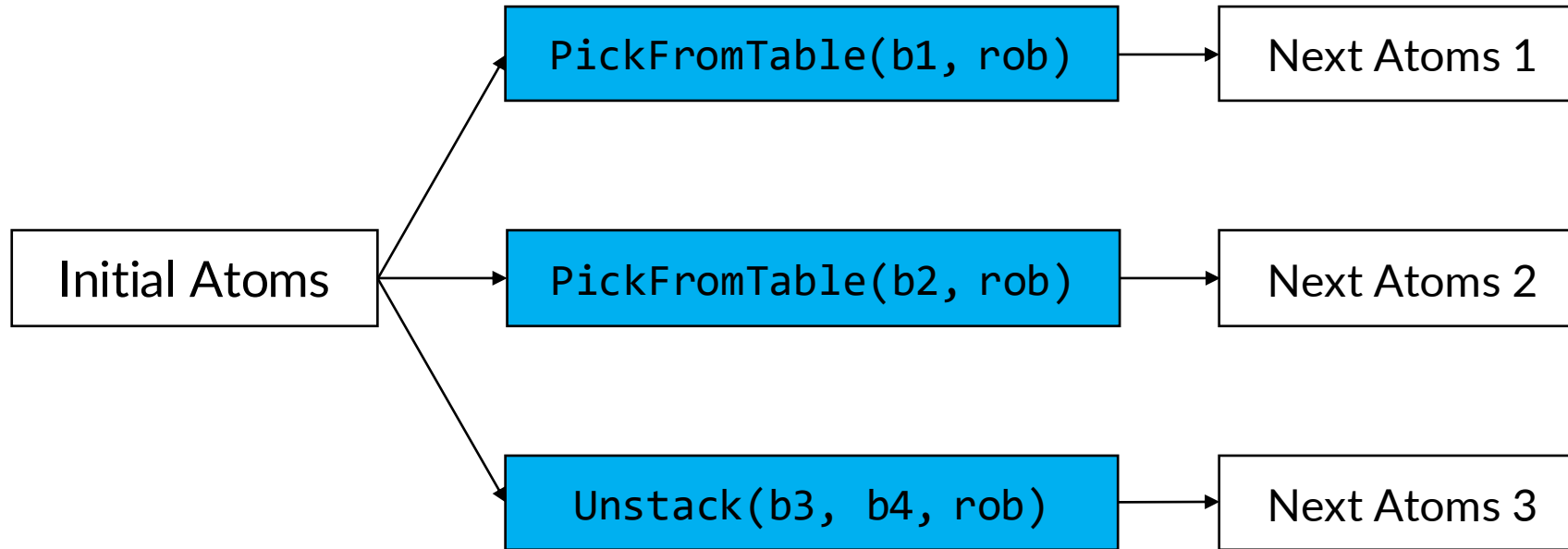
Parameters: [b1 , ]

Preconditions: {GripperOpen() ,
OnTable(b1)}

Add effects: {Holding(b1)}

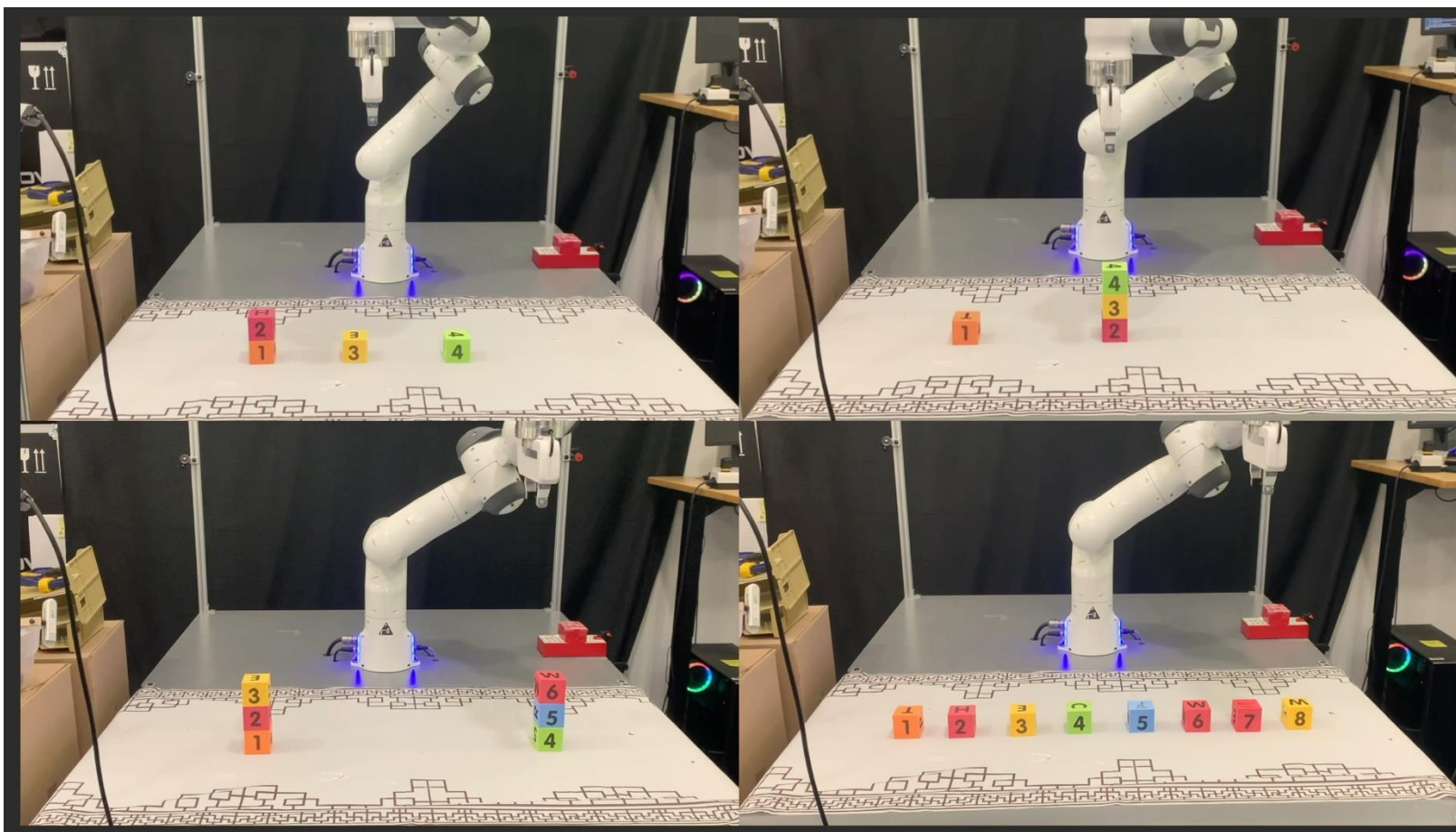
Delete effects: {GripperOpen() ,
OnTable(b1)}

Recall: Task Planning as Search

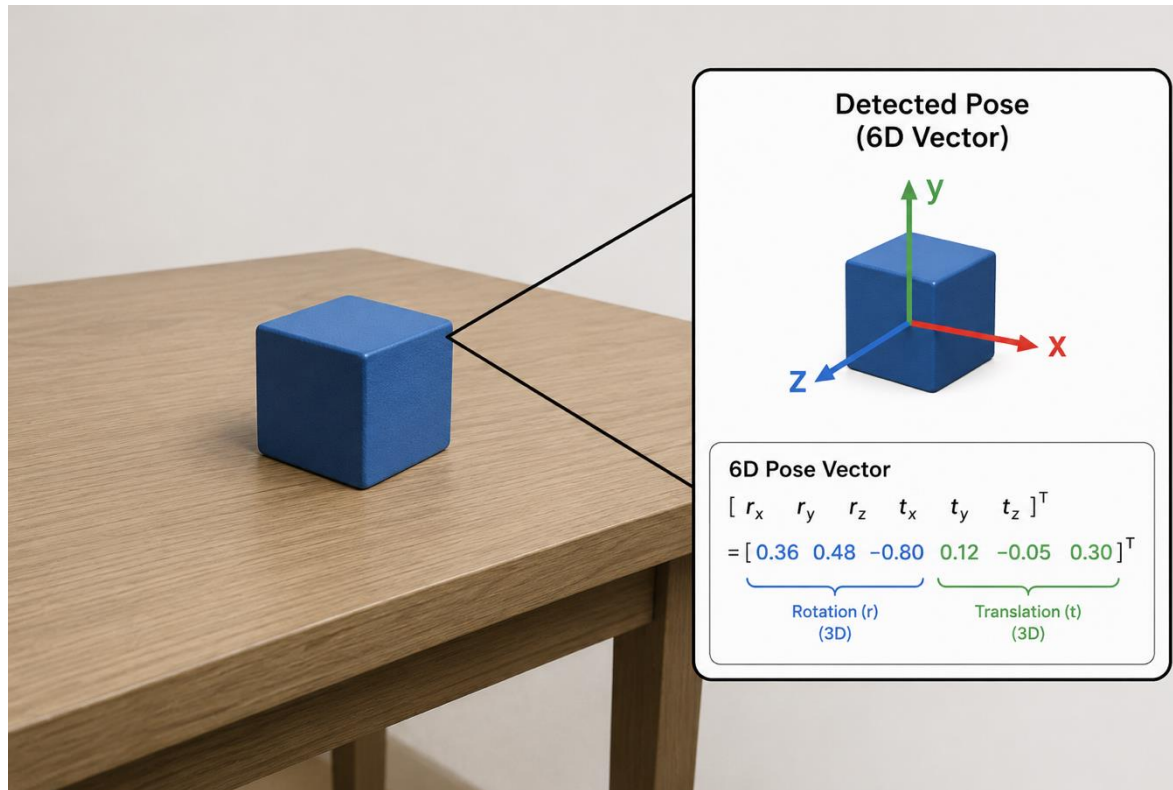


An (partial) transition model

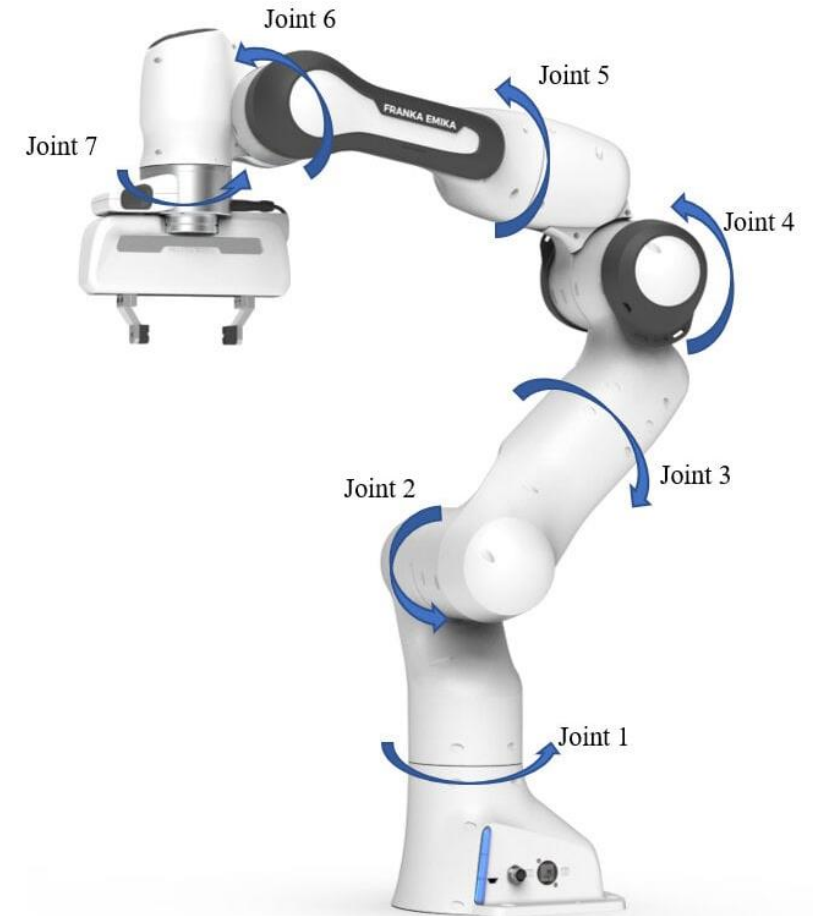
Task Planning on a Robot?



Robotics: Continuous States and Actions

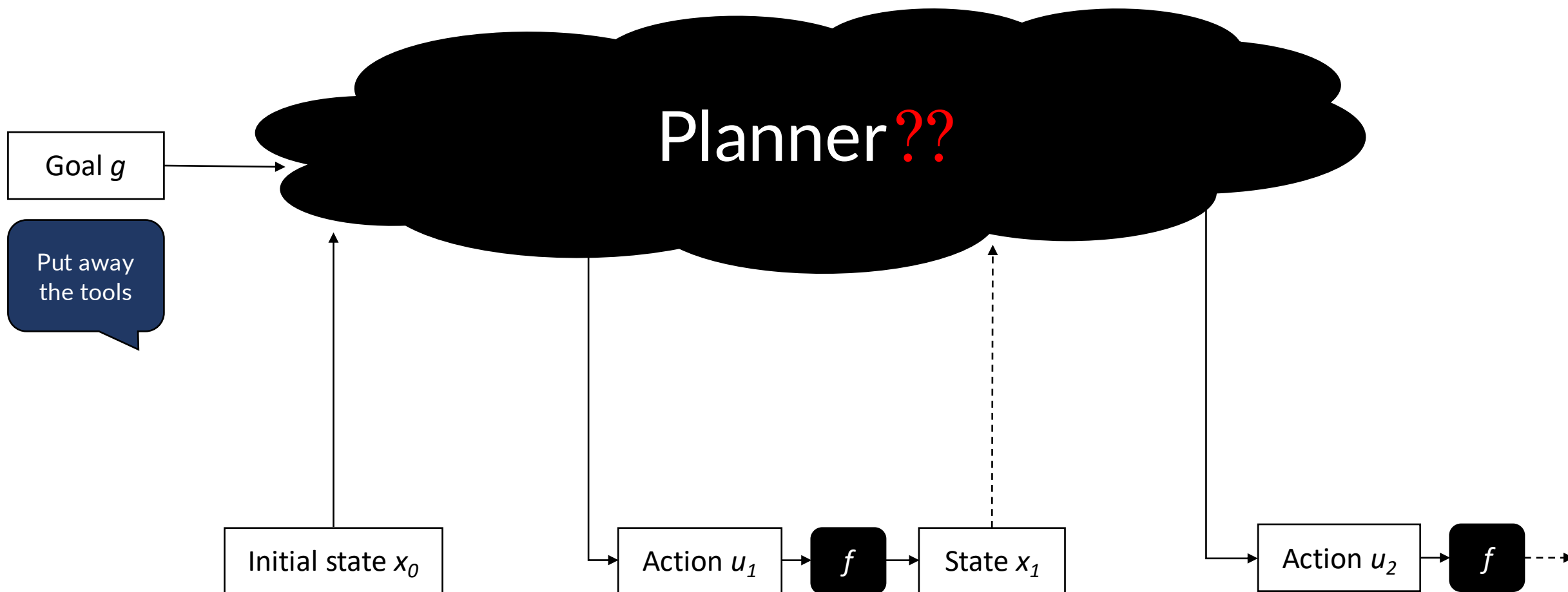


ChatGPT

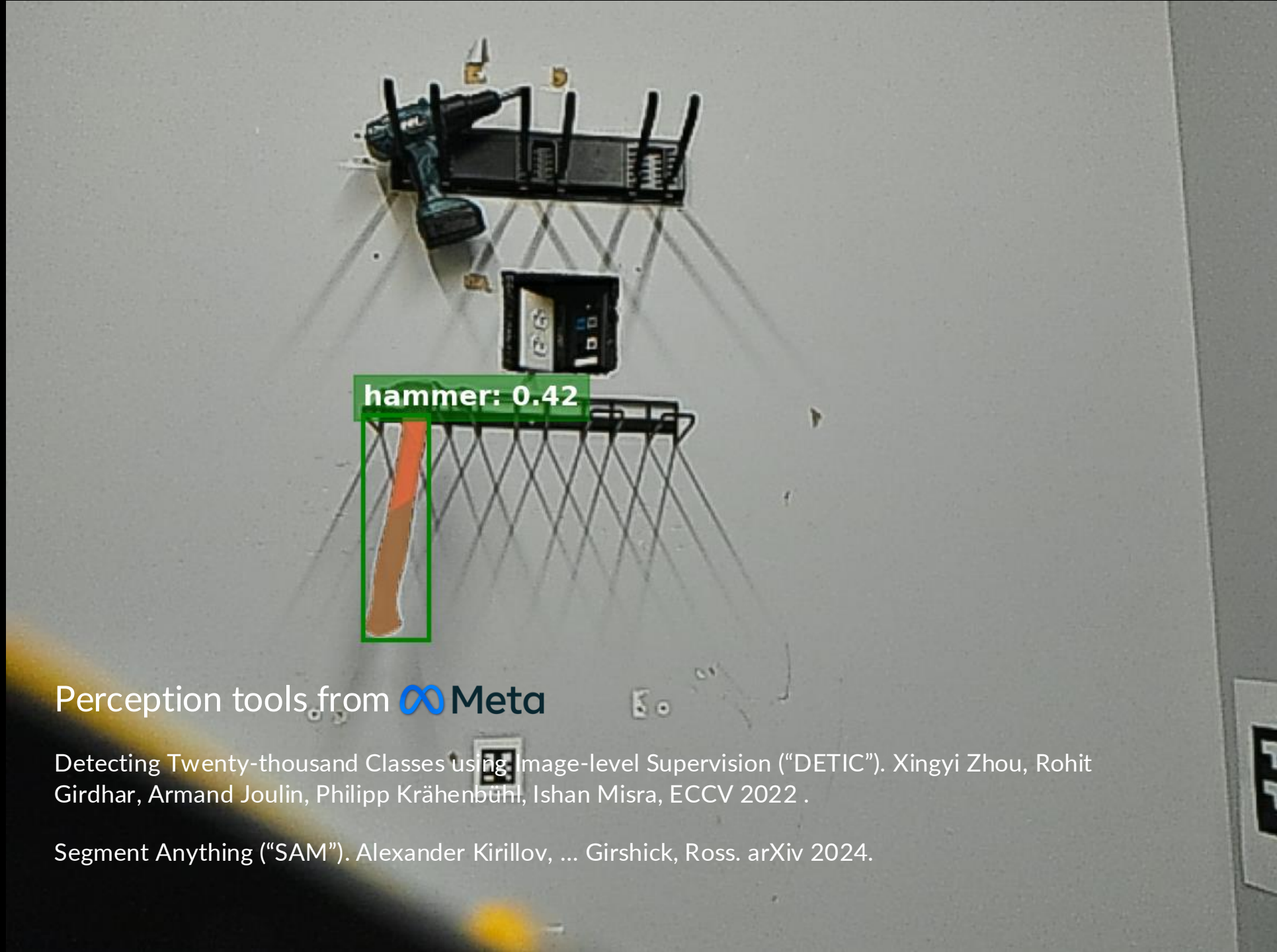


A yellow Boston Dynamics Spot robot is positioned in the center of the frame, facing away from the camera and slightly to the right. The robot has a black arm and gripper. In the background, a white wall features a metal tool rack with a red-handled tool hanging from it. A white electrical outlet is visible on the wall. The floor is dark grey carpet. A blue speech bubble with white text is located in the upper right corner.

Hey Spot, put
away the tools!







Perception tools from Meta

Detecting Twenty-thousand Classes using Image-level Supervision (“DETIC”). Xingyi Zhou, Rohit Girdhar, Armand Joulin, Philipp Krähenbühl, Ishan Misra, ECCV 2022 .

Segment Anything (“SAM”). Alexander Kirillov, ... Girshick, Ross. arXiv 2024.

Object-Centric States



id	type	x	y	height	...
hammer	tool	7.44	5.25	0.32	...



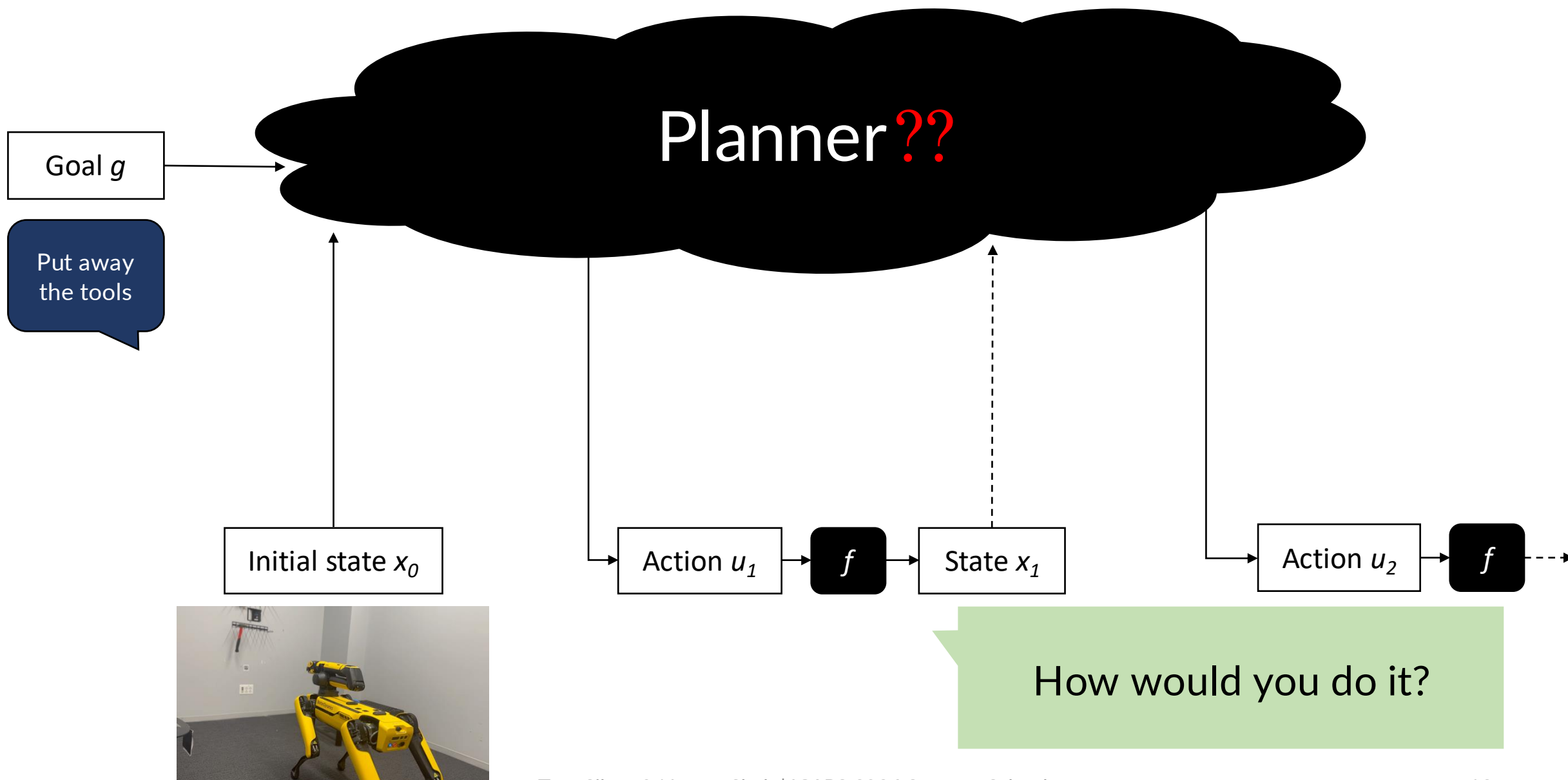
id	type	x	y	radius	...
bucket	container	0.93	6.52	0.28	...

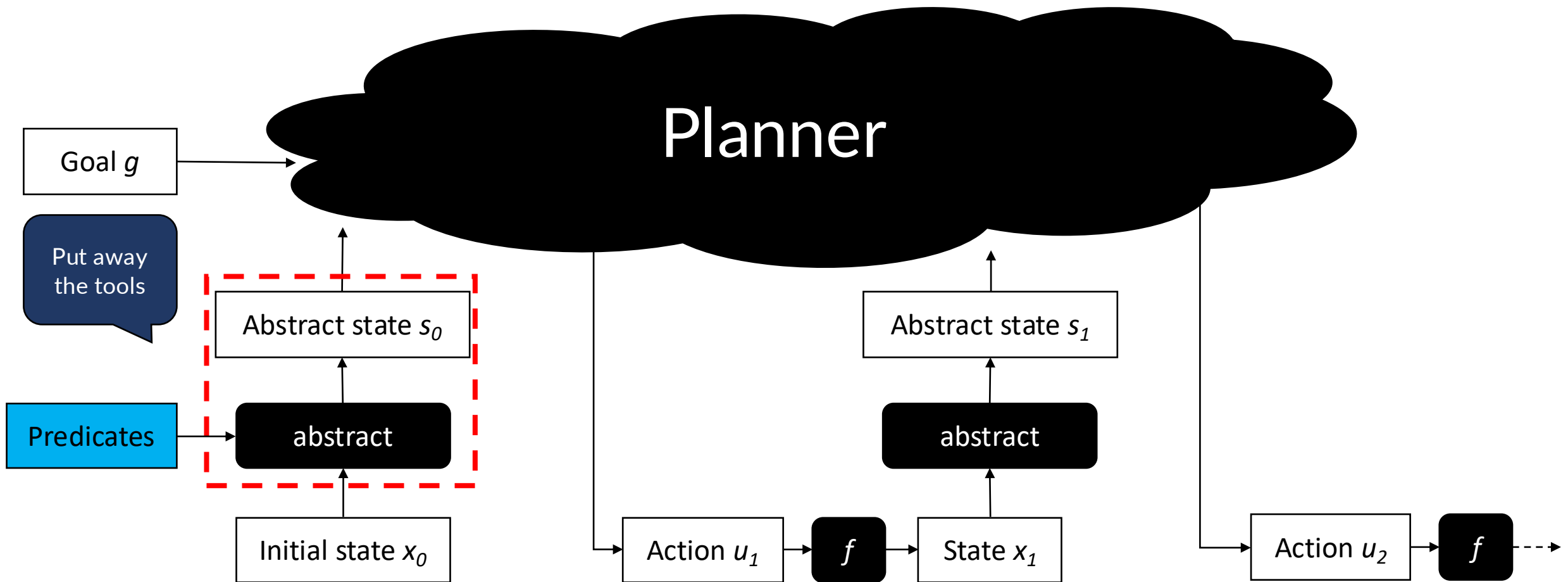


id	type	x	y	gripper	...
Spot	robot	5.25	4.91	0.04	...

...

...





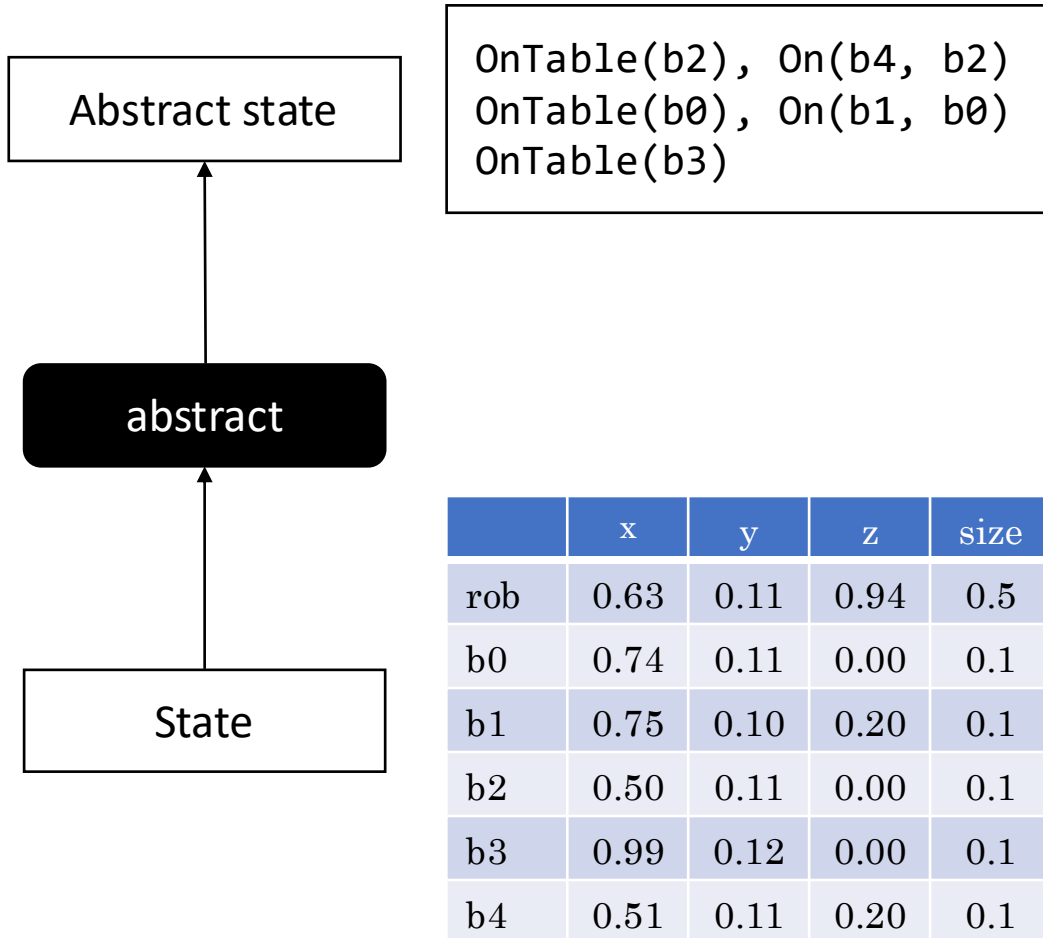
	id	type	x	y	height	...
	hammer	tool	7.44	5.25	0.32	...

	id	type	x	y	radius	...
	bucket	container	0.93	6.52	0.28	...

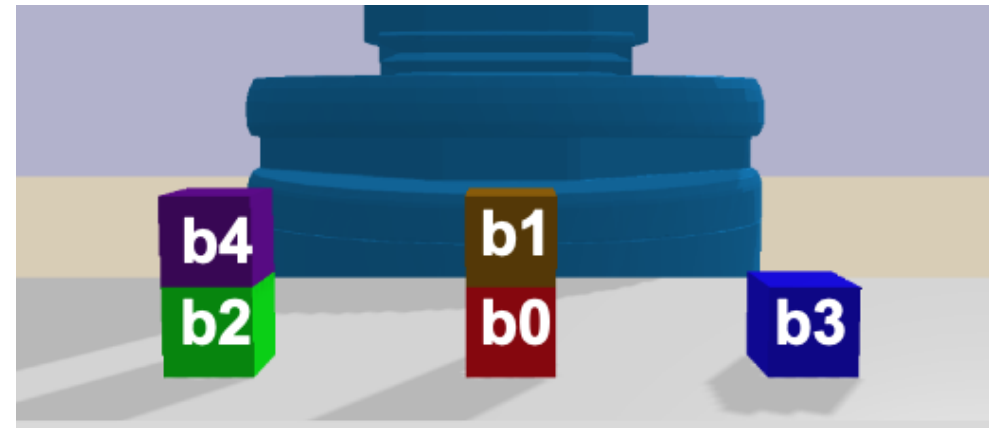
	id	type	x	y	gripper	...
	Spot	robot	5.25	4.91	0.04	...

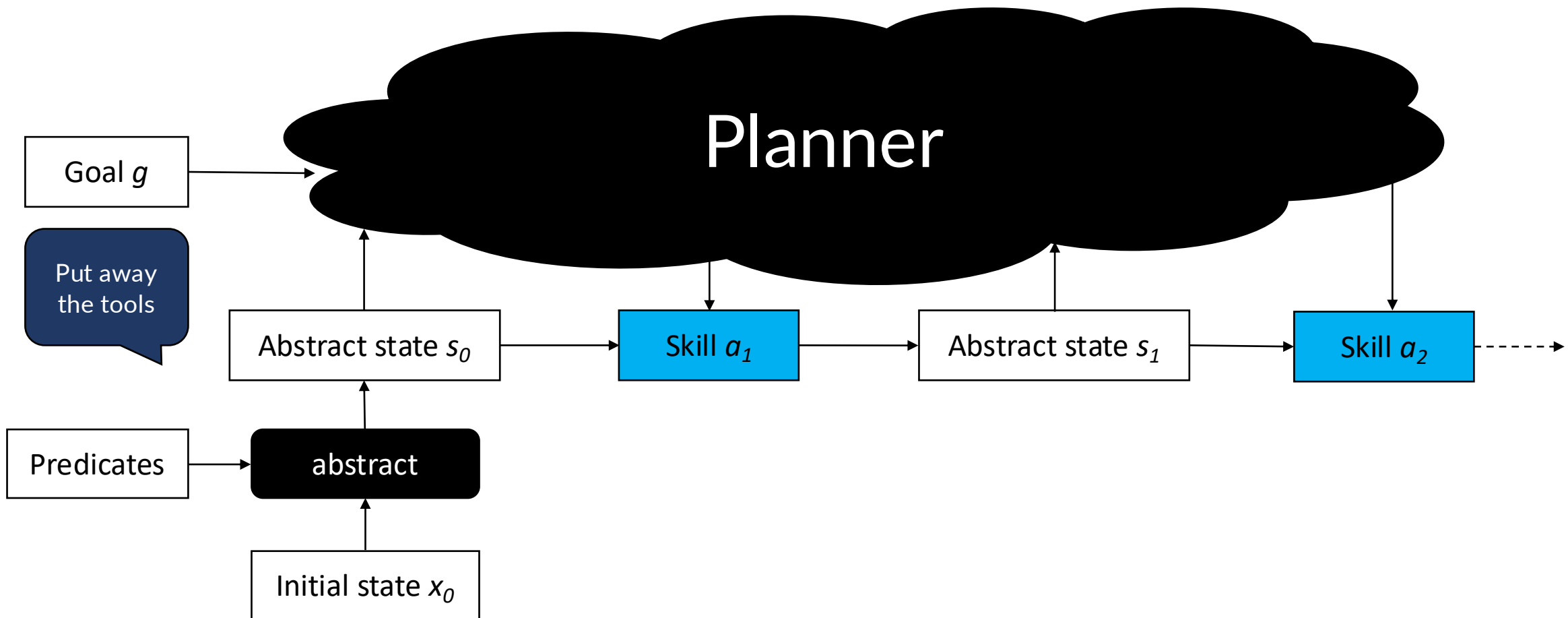
State abstraction via predicates

Predicates: Relational Classifiers over Object States



```
def classifyOnTable(state, ?block):  
    state[?block].z < 1e-5  
  
def classifyOn(state, ?top, ?bot):  
    (state[?top].z - state[?bot].z -  
     state[?bot].size) < 1e-5 &  
    (state[?top].x - state[?bot].x) < 1e-5 &  
    (state[?top].y - state[?bot].y) < 1e-5 &
```





	id	type	x	y	height	...
	hammer	tool	7.44	5.25	0.32	...

	id	type	x	y	radius	...
	bucket	container	0.93	6.52	0.28	...

	id	type	x	y	gripper	...
	Spot	robot	5.25	4.91	0.04	...

Action abstraction via skills

Skill Operators (STRIPS)

Arguments

List of typed variables

Preconditions

What must be true in order to use this operator?

Add/Delete Effects

How is the abstract state changed by this operator?

Operator-PickFromTable:

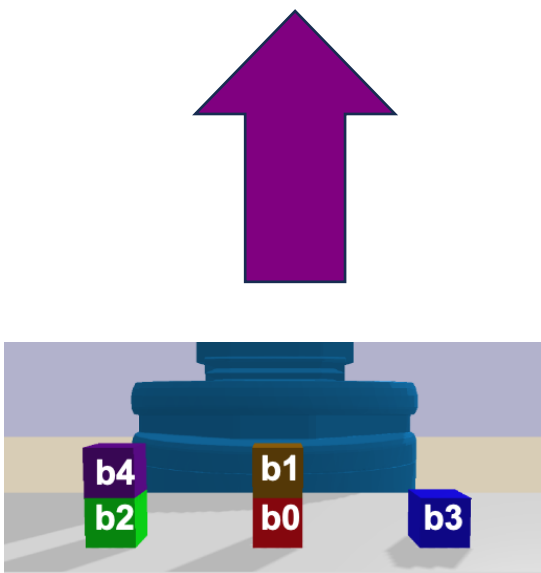
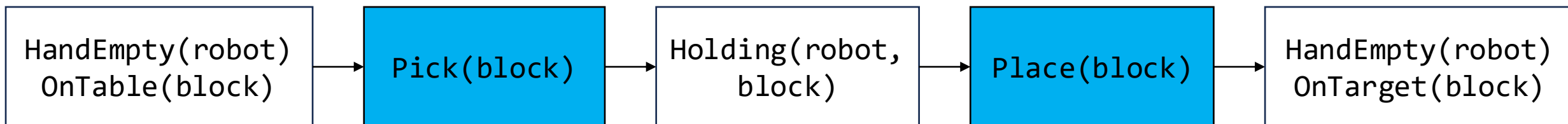
Arguments: [**?b** - block, **?r** - robot]

Preconditions: {GripperOpen(**?r**),
OnTable(**?b**)}

Add effects: {Holding(**?b**)}

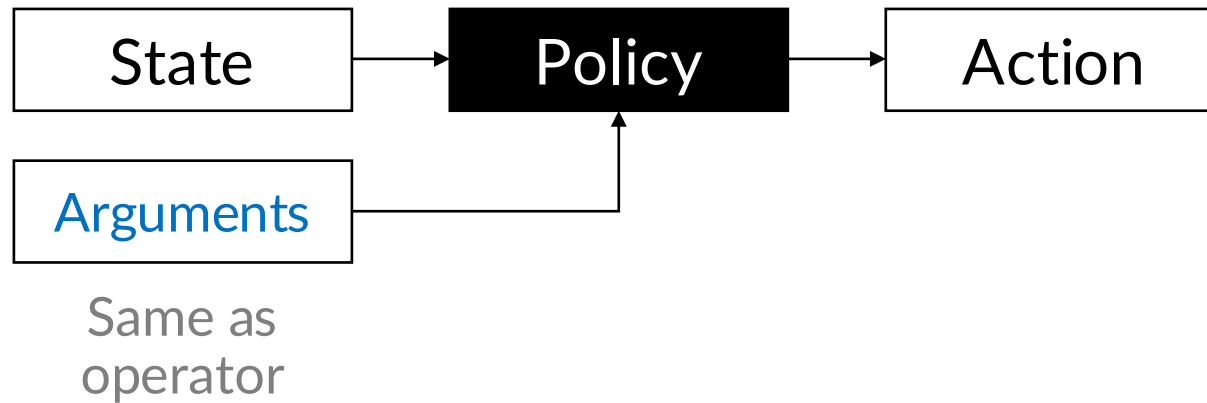
Delete effects: {GripperOpen(**?r**),
OnTable(**?b**)}

How to “Execute” an Abstract Plan?



How would you do it?

Implementing Action Abstraction with Relational Policies



```
def policyPickFromTable(state, ?b, ?r):  
    dx = (state[?b].x - state[?r].x)  
    dy = (state[?b].y - state[?r].y)  
    dz = (state[?b].z - state[?r].z)  
    return [dx, dy, dz]
```

Simplified example

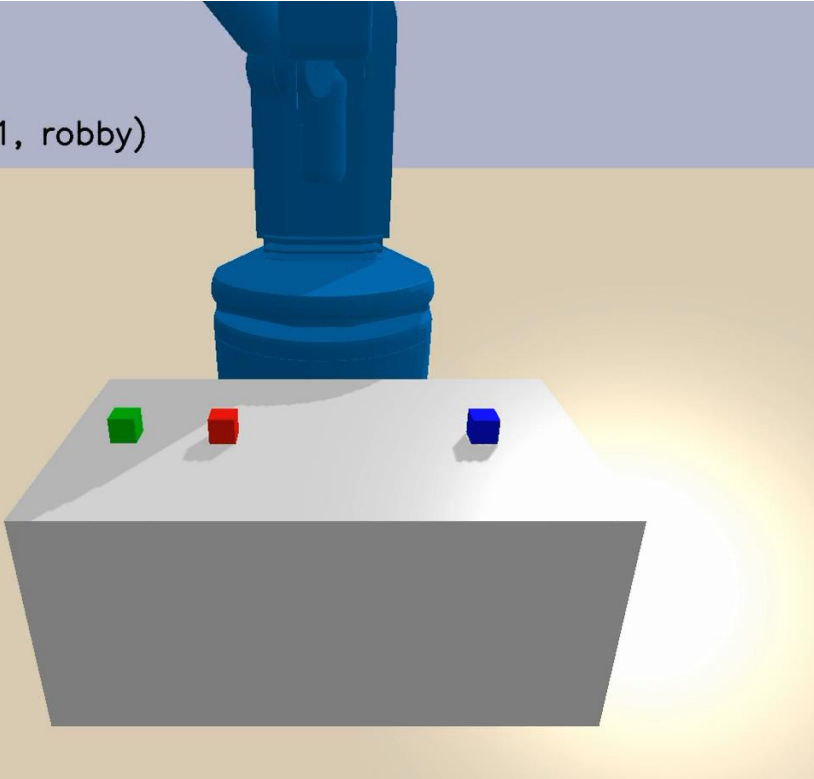
The policy should *achieve* the operator effects when the operator preconditions hold

Example Policy Executions

PickFromTable(block1, robby)

Abstract State:

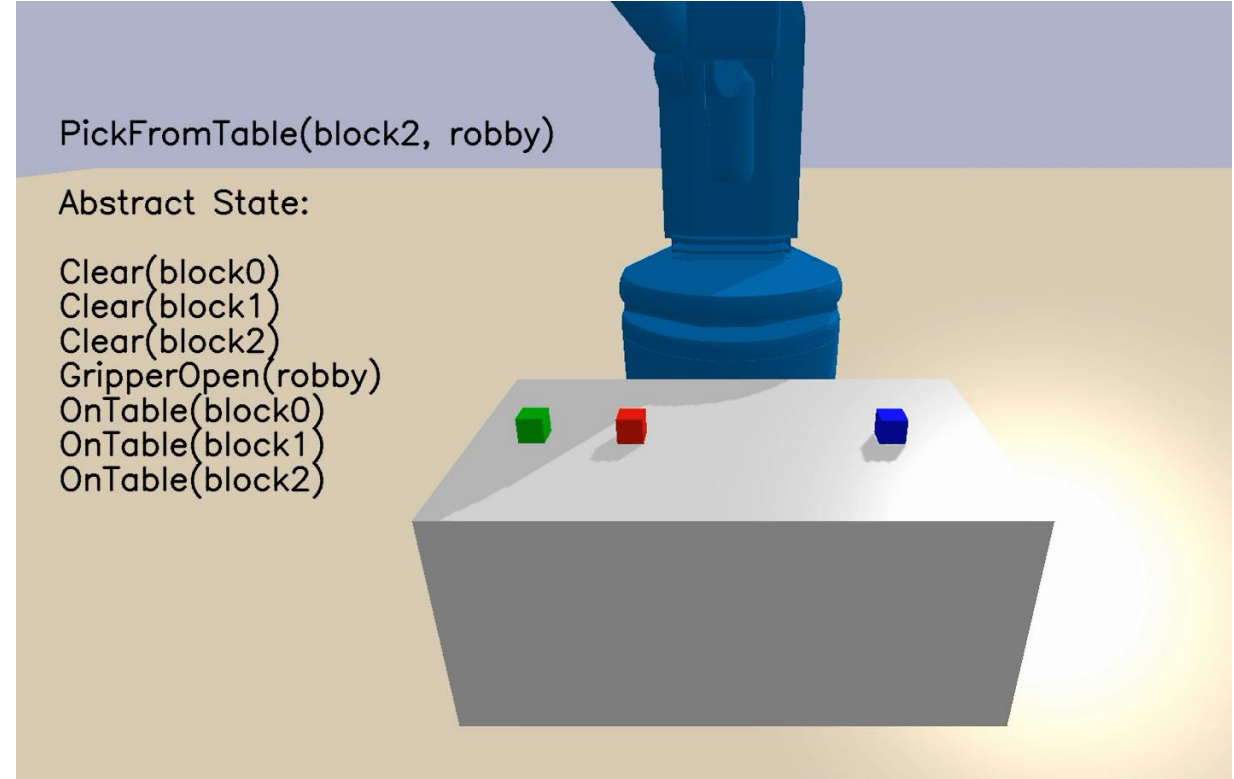
```
Clear(block0)
Clear(block1)
Clear(block2)
GripperOpen(robby)
OnTable(block0)
OnTable(block1)
OnTable(block2)
```



PickFromTable(block2, robby)

Abstract State:

```
Clear(block0)
Clear(block1)
Clear(block2)
GripperOpen(robby)
OnTable(block0)
OnTable(block1)
OnTable(block2)
```

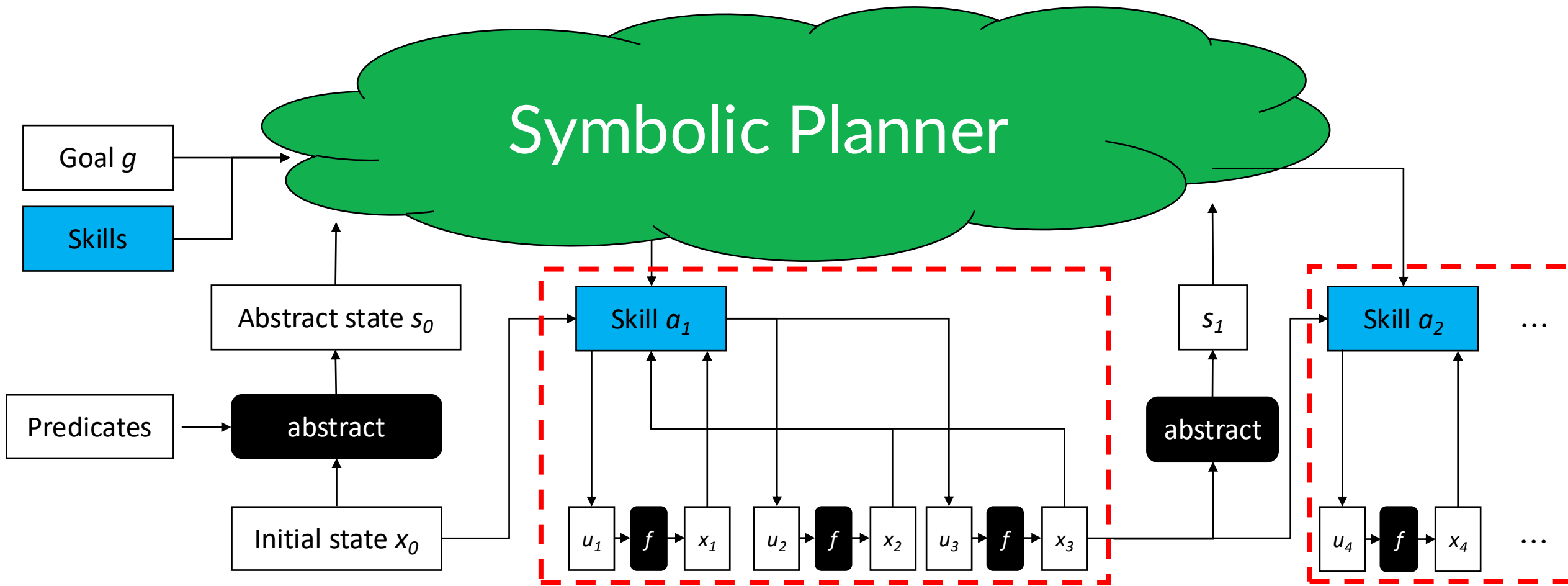


Skills: abstract actions that bring the robot from one abstract state to another

What abstract state transition?

How should I get there?

A skill has an **operator** and a **policy**

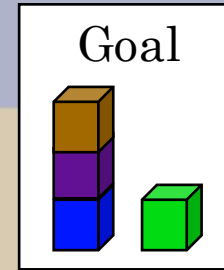


Action abstraction via skills

Unstack(block4, block3, robby)

Abstract State:

Clear(block4)
GripperOpen(robby)
On(block1, block0)
On(block2, block1)
On(block3, block2)
On(block4, block3)
OnTable(block0)



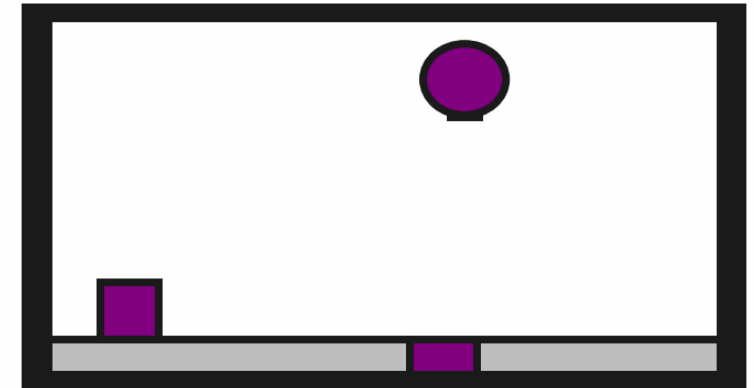
Domain for Tonight's Lab

State space: Robot config, block pose (8D)

Action space: Pose change, vacuum (5D)

Transition function: Apply action but disallow collisions (no change)

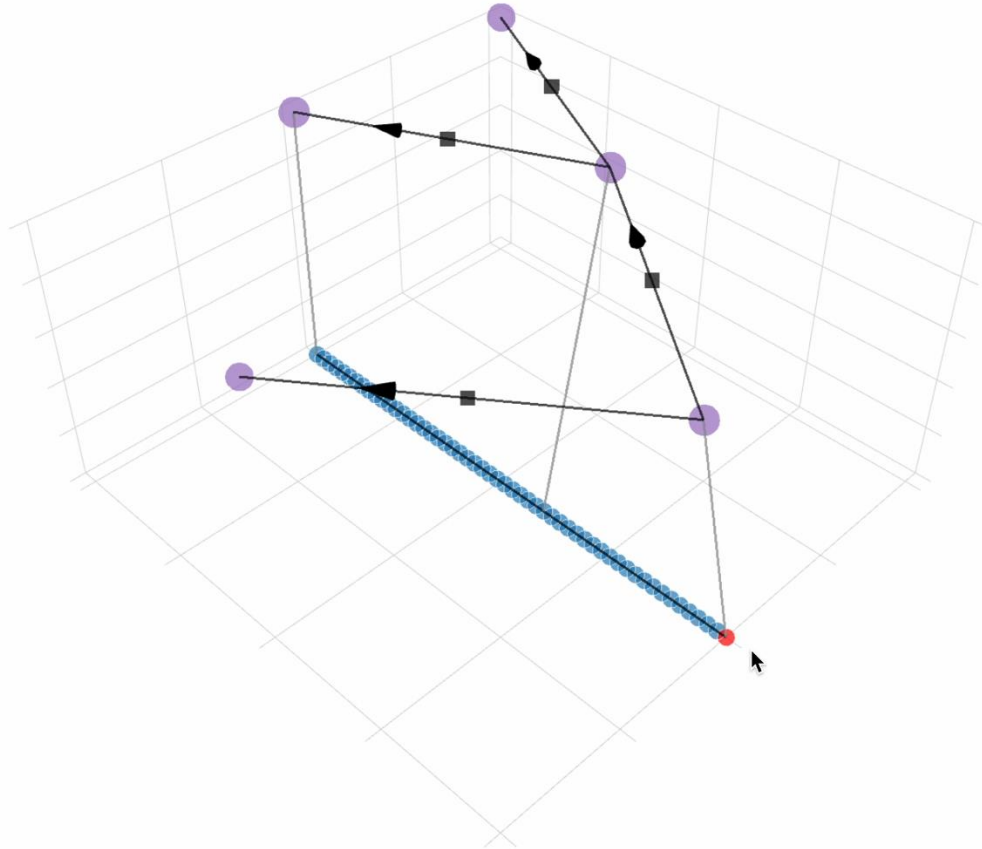
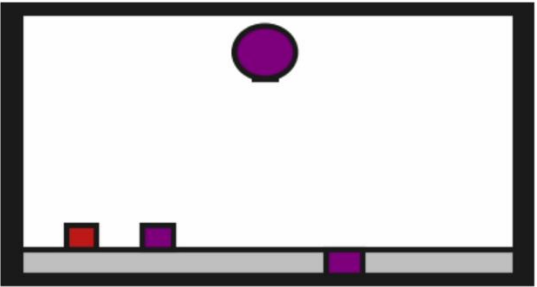
Goal: Block on target region



- Concrete states
- Abstract states
- Abstraction

Selected Node: x:0

Visualization:



Timeline: 1 / 54



< Prev Next > Play Pause

Speed (ms): 200

Red = current; darker = visited.

Can hold prev/next to auto-step.

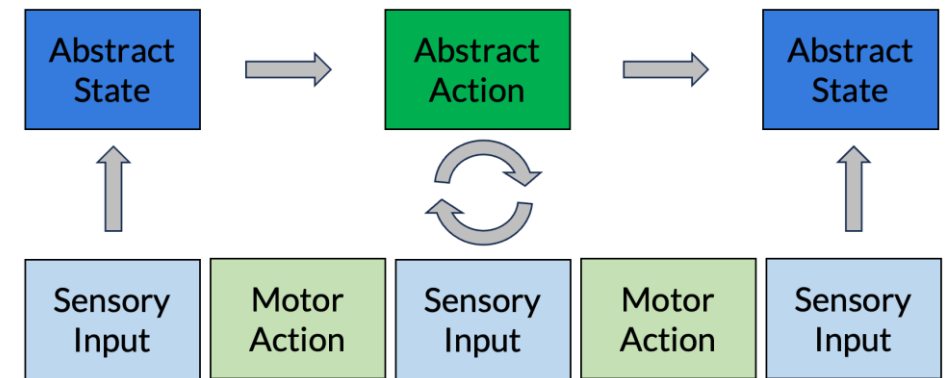
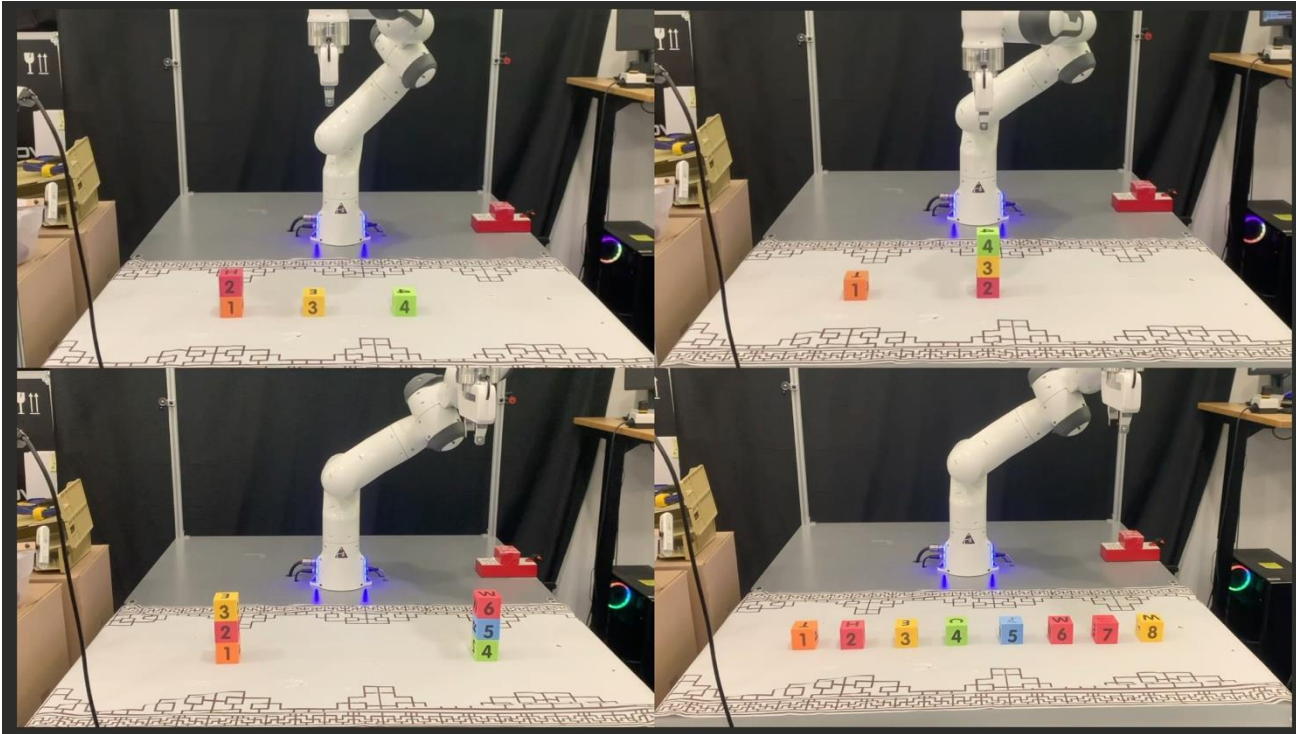
Zoom

+ - Reset view

Buttons zoom past the scroll-wheel limit.

Nodes: 59

Task Planning on a Robot?





FOCUS

TYPE A SENTENCE

GOTHRU

PRECONDITION:

Near Door

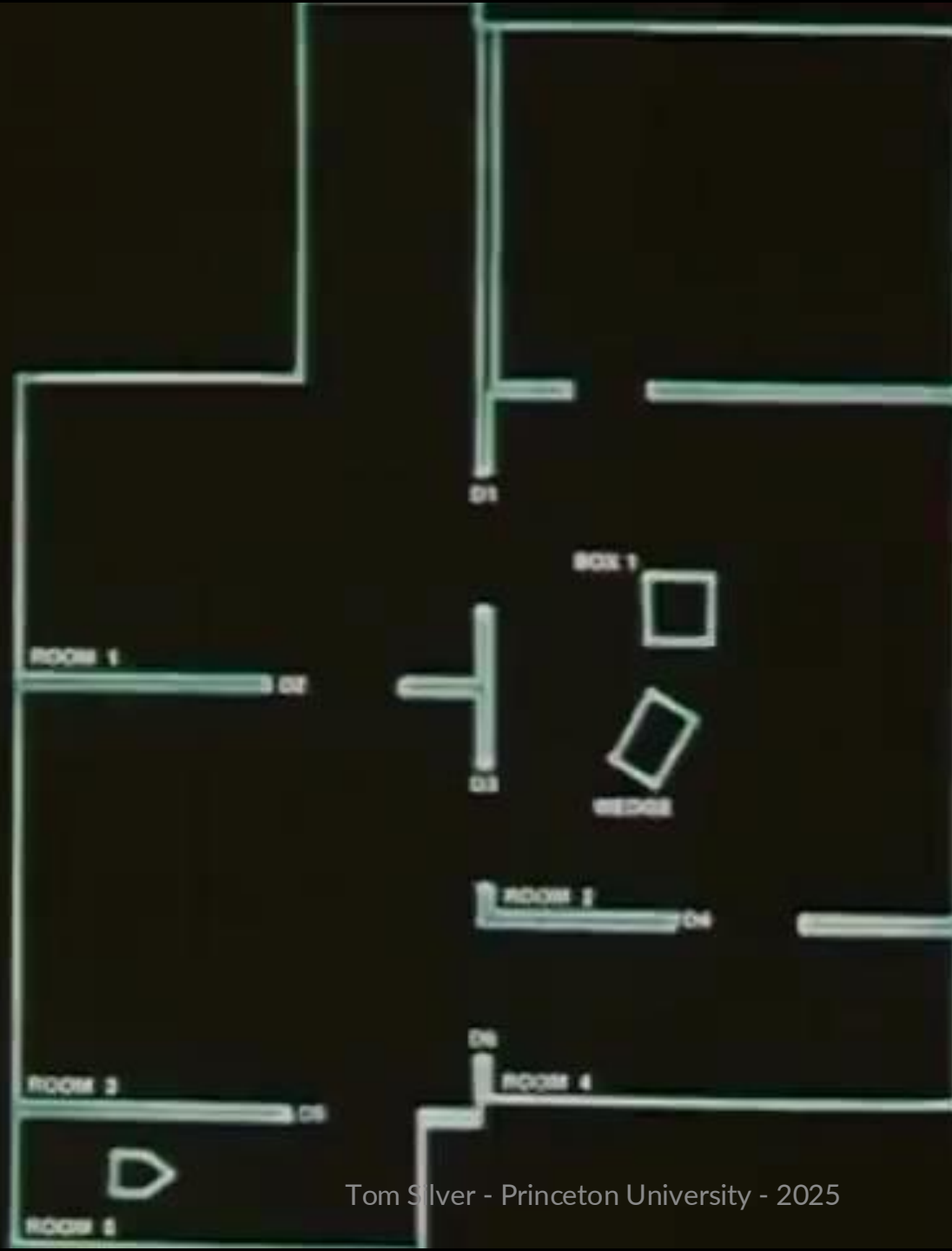
EFFECTS:

Will Be In Adjacent Room

START



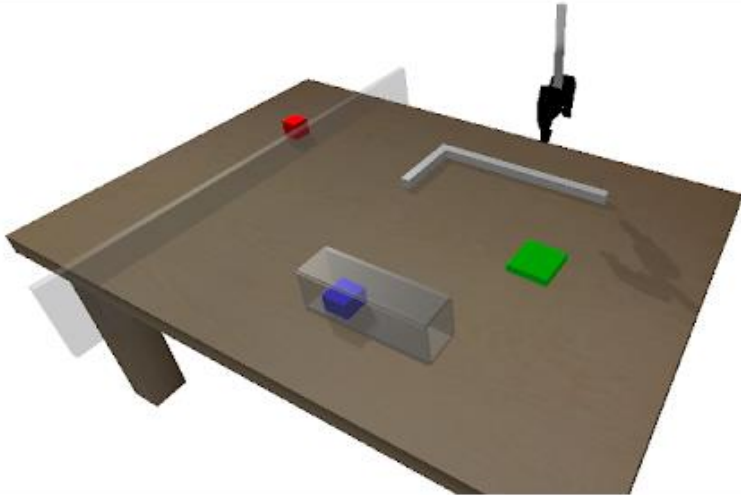
GOAL





The abstractions might be liars...





“Deep Affordance Foresight: Planning Through What Can Be Done in the Future.” Danfei Xu, Ajay Mandlekar, Roberto Martin-Martin , Yuke Zhu, Silvio Savarese and Li Fei-Fei. ICRA 2021.

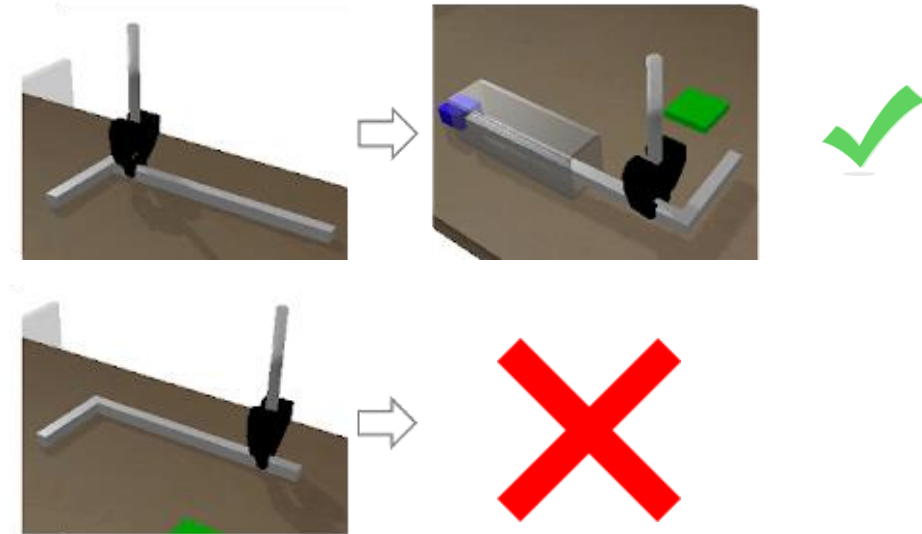
Operator-PushOutOfTube:

Arguments: [ , ]

Preconditions: {Holding(),
InTube()}

Add effects: {OutOfTube()}

Delete effects: {InTube()}



What should we do?

Possible Conclusions from this Example

1. Insufficient predicates → learn new predicates

HoldingBottom, HoldingTop, etc.

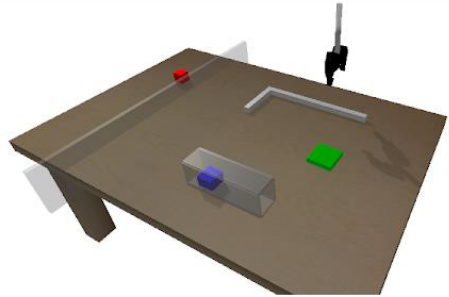
2. Insufficient policies → learn better policies

Put down the tool and regrasp if needed

3. Insufficient planner → be less trusting of the abstractions

View abstractions as *guidance* for low-level planning

Parameterized Skill Policies



Operator-PickTool:

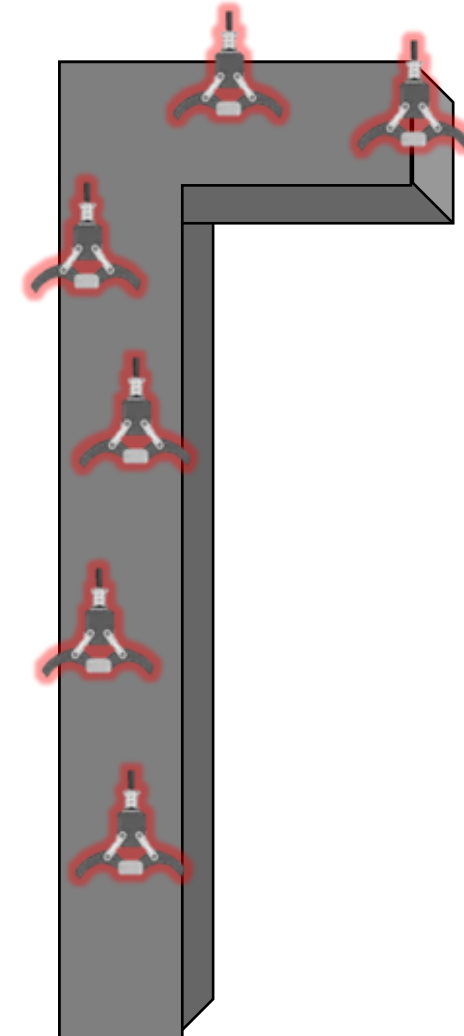
Arguments: [, ]

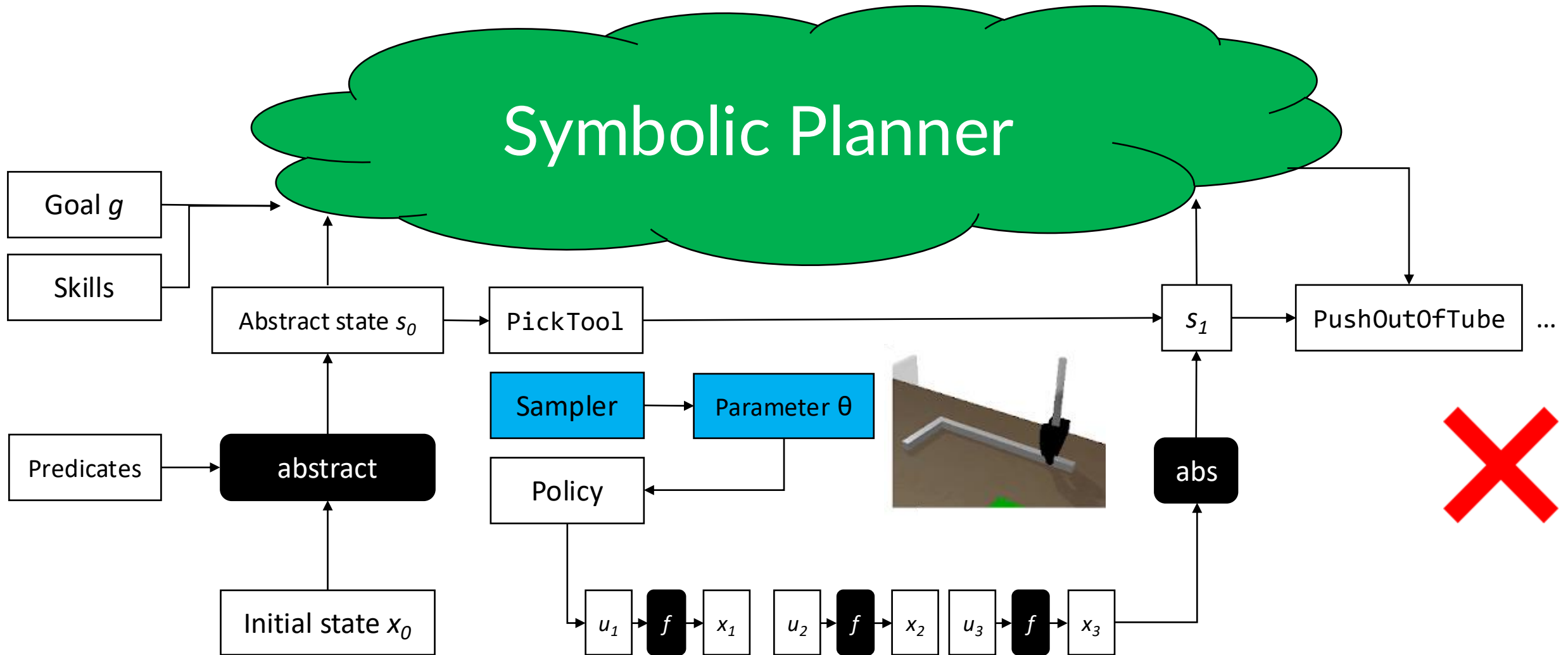
Preconditions: {GripperOpen()}

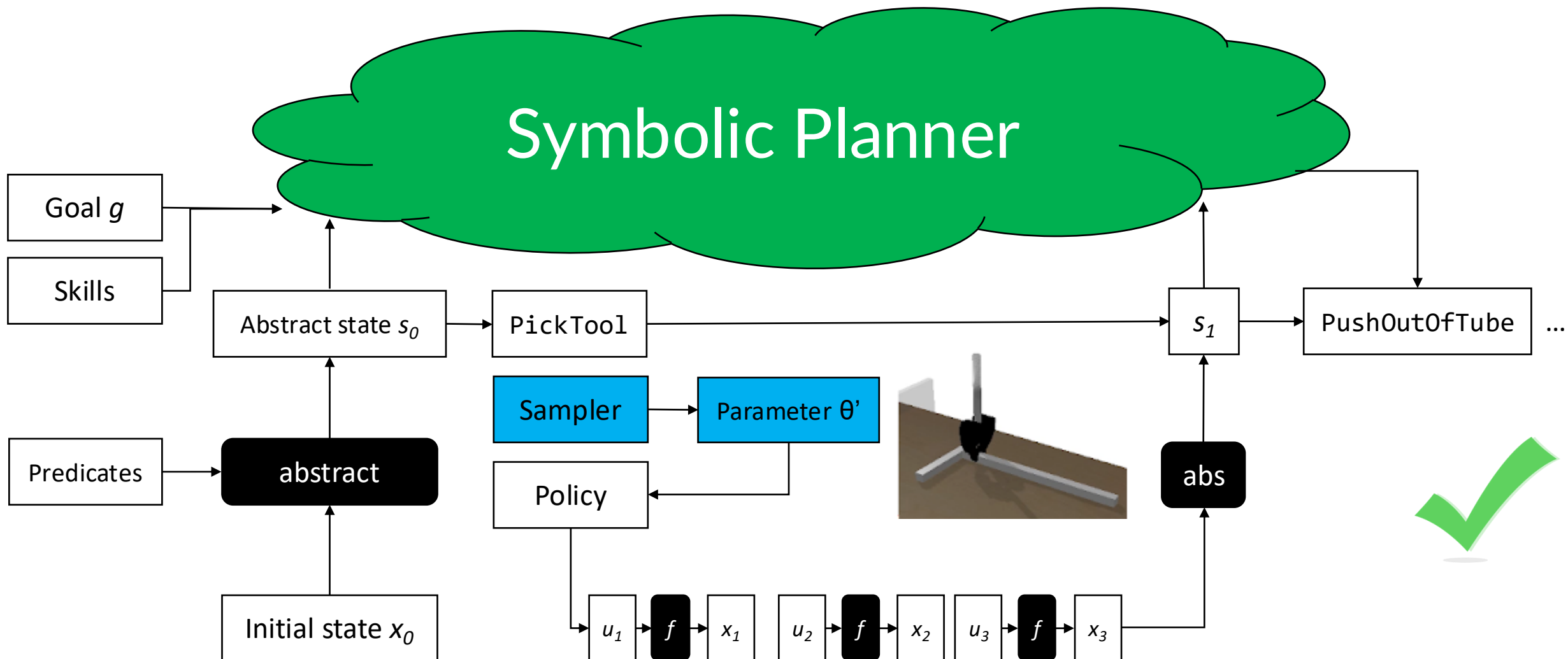
Add effects: {Holding(, )}

Delete effects: {GripperOpen()}

Different Parameters

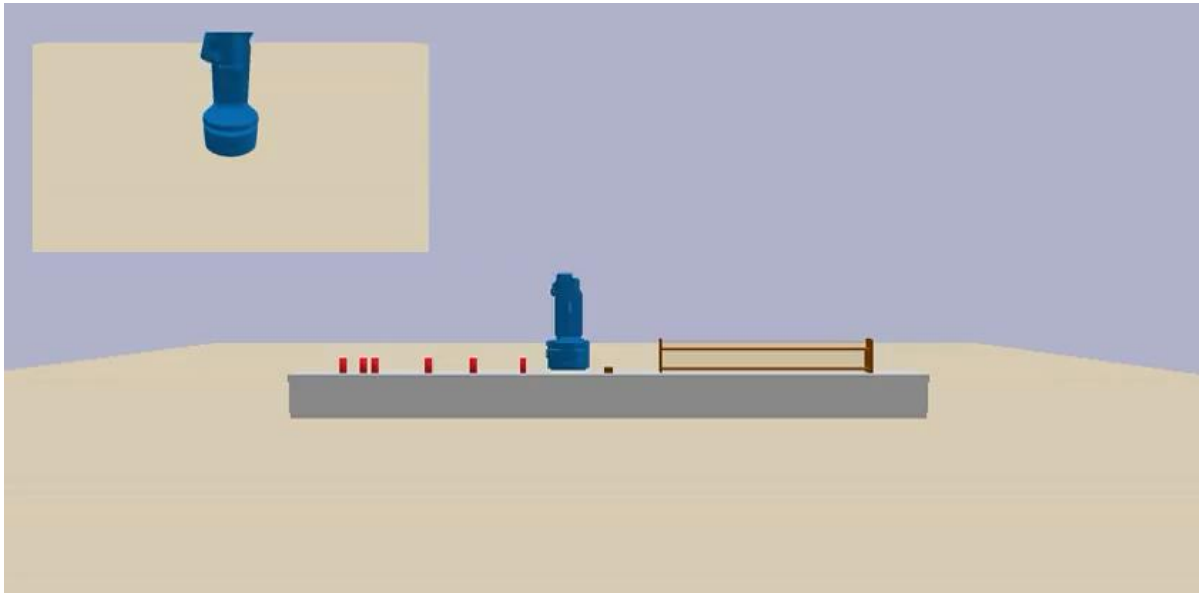






Let's Be Clear About How Terrible Backtracking Sampling Can Be

`num_sampling_attempts_per_step`: the number of samples to draw at each step before backtracking

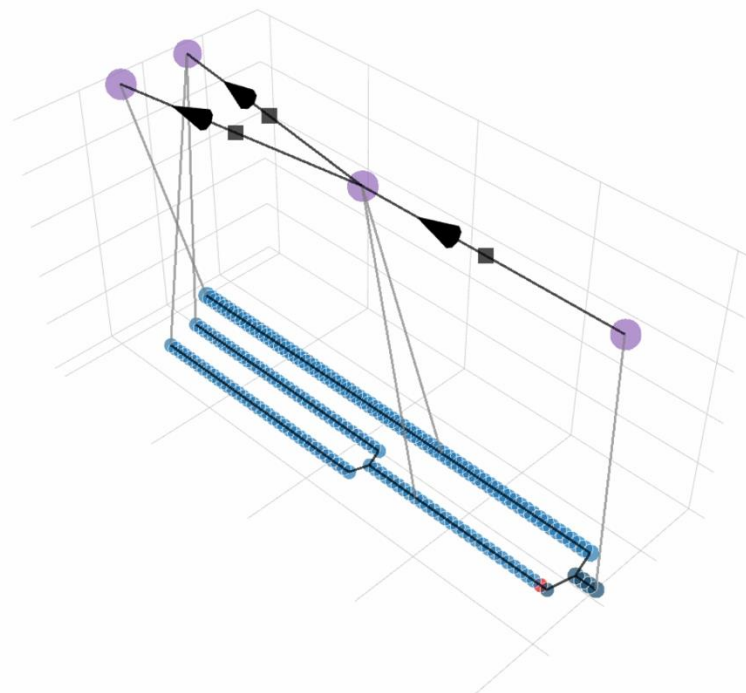


Pick (top or side grasp)
Wash
Dry
Paint
Place in box/shelf

What's the time complexity of backtracking search?

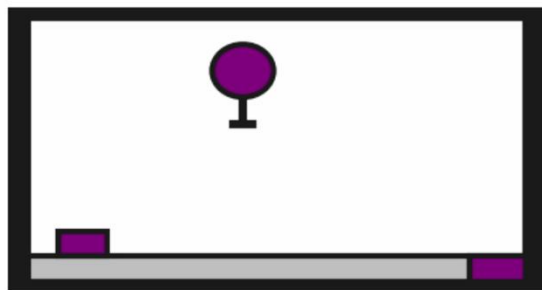
- Concrete states
- Abstract states
- Abstraction

`num_sampling_attempts_per_step = 2`



Selected Node: x:5

Visualization:



Timeline: 6 / 178



< Prev Next > **Play** Pause

Speed (ms): 200

Red = current; darker = visited.

Can hold prev/next to auto-step.

Zoom

+ - Reset view

Buttons zoom past the scroll-wheel limit.

Nodes: 182

The abstractions may be
pathological liars...

An abstract plan may not be
refinable at all!

Coffee Domain

Abstract Plan

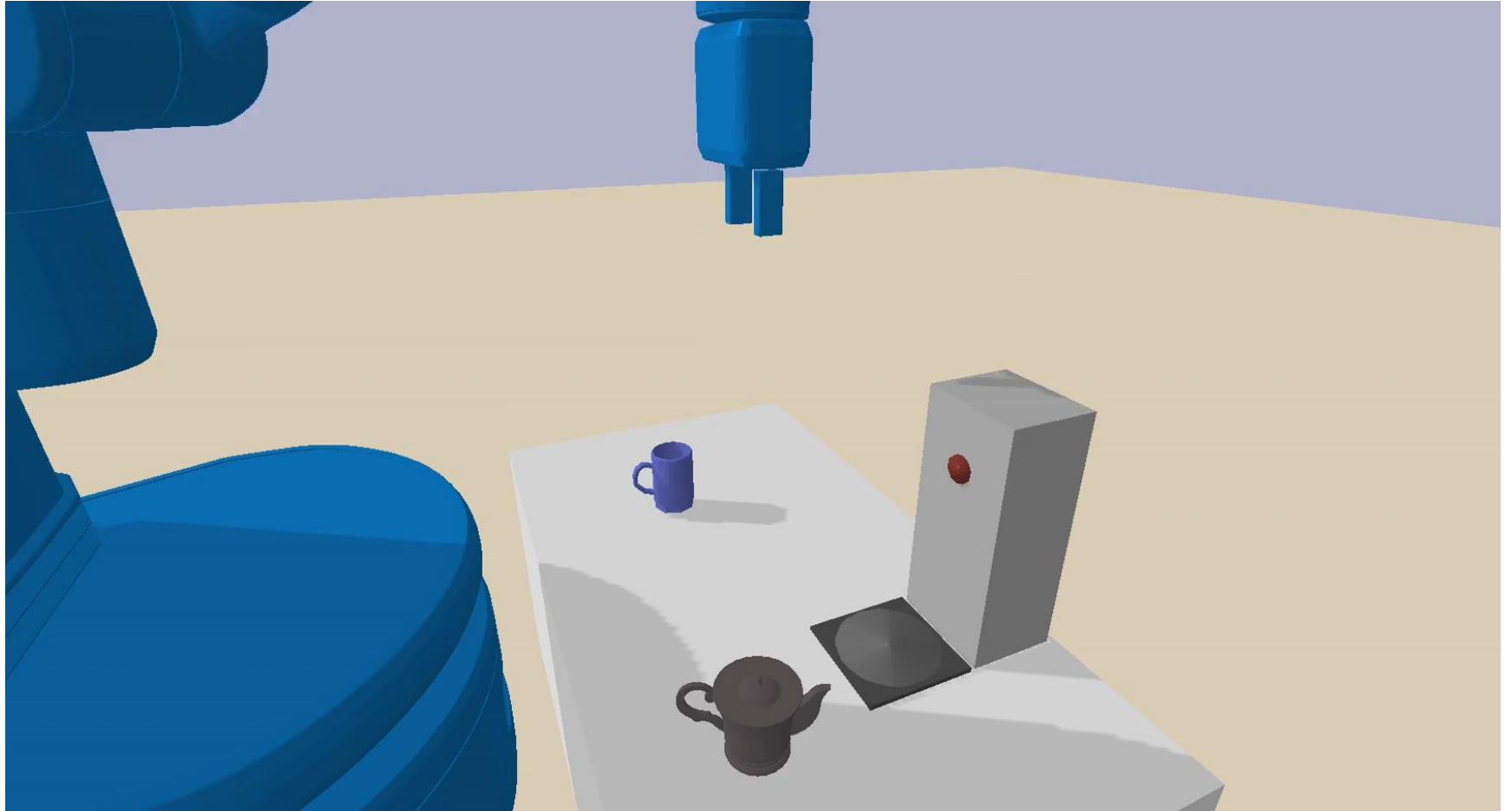
Grasp handle

Place on plate

Turn plate on

Pick up pot

Pour into cup



We need a different
abstract plan!

Coffee Domain

Abstract Plan



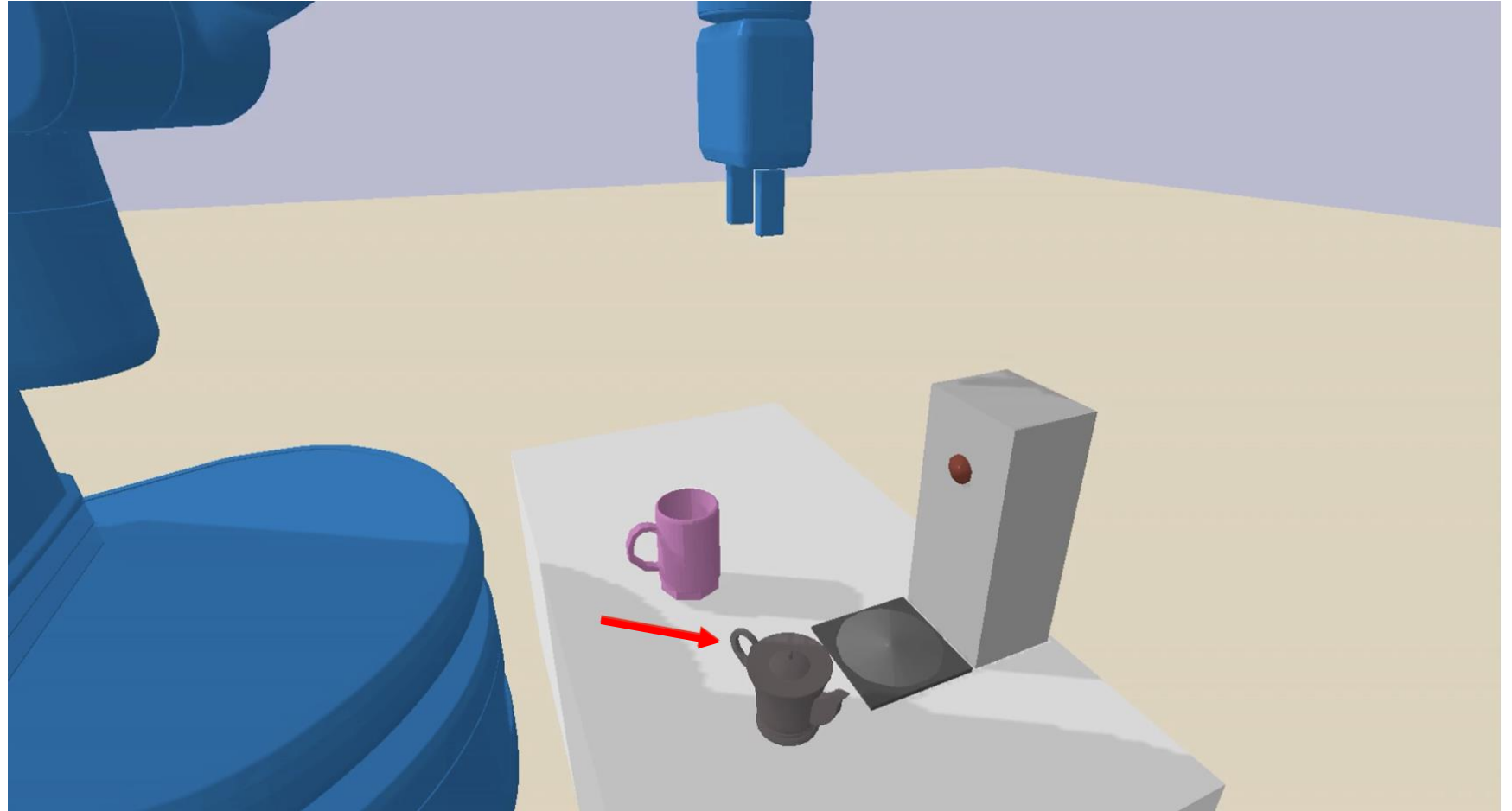
Grasp handle

Place on plate

Turn plate on

Pick up pot

Pour into cup



Coffee Domain

Abstract Plan

Rotate pot

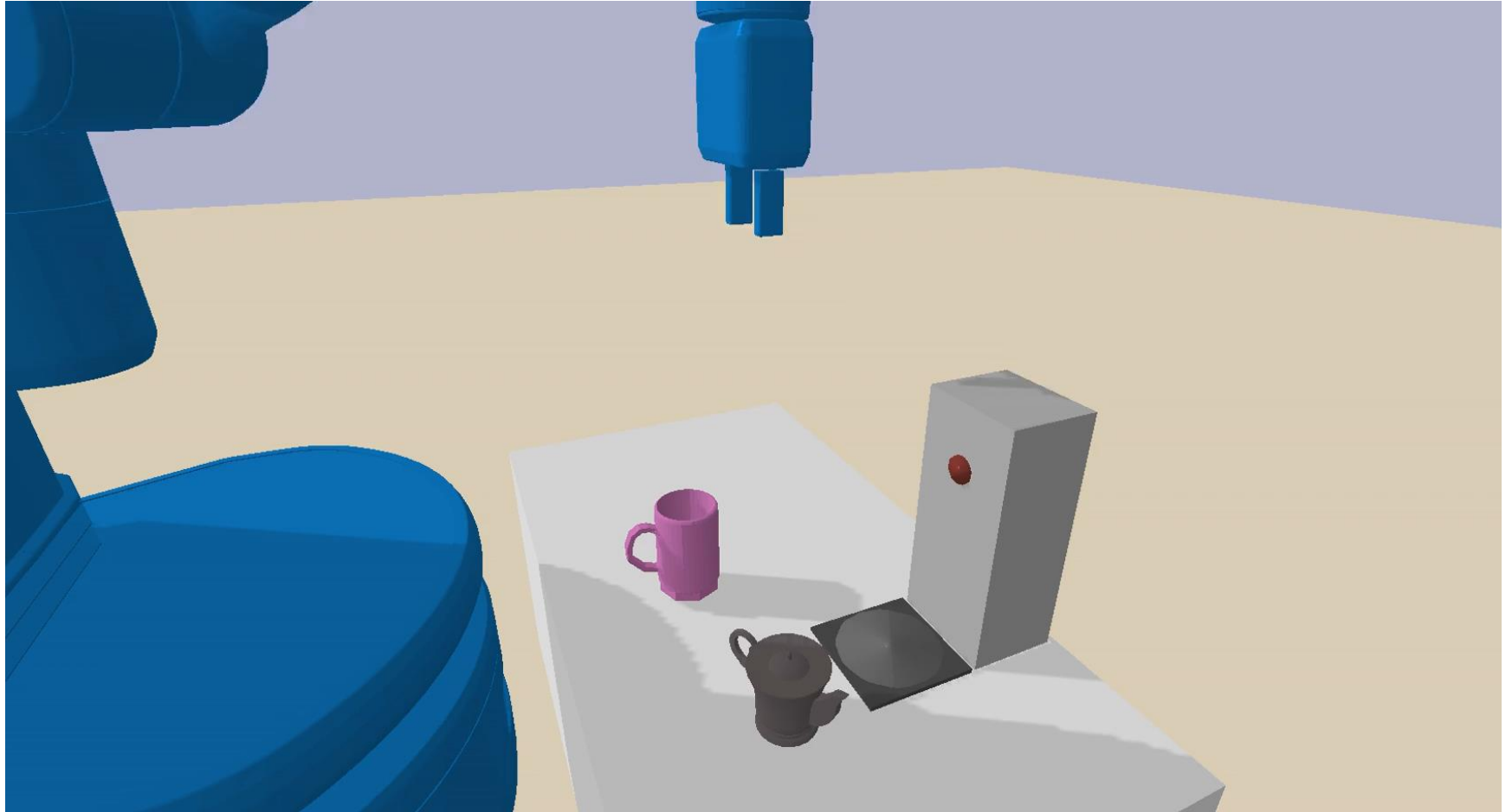
Grasp handle

Place on plate

Turn plate on

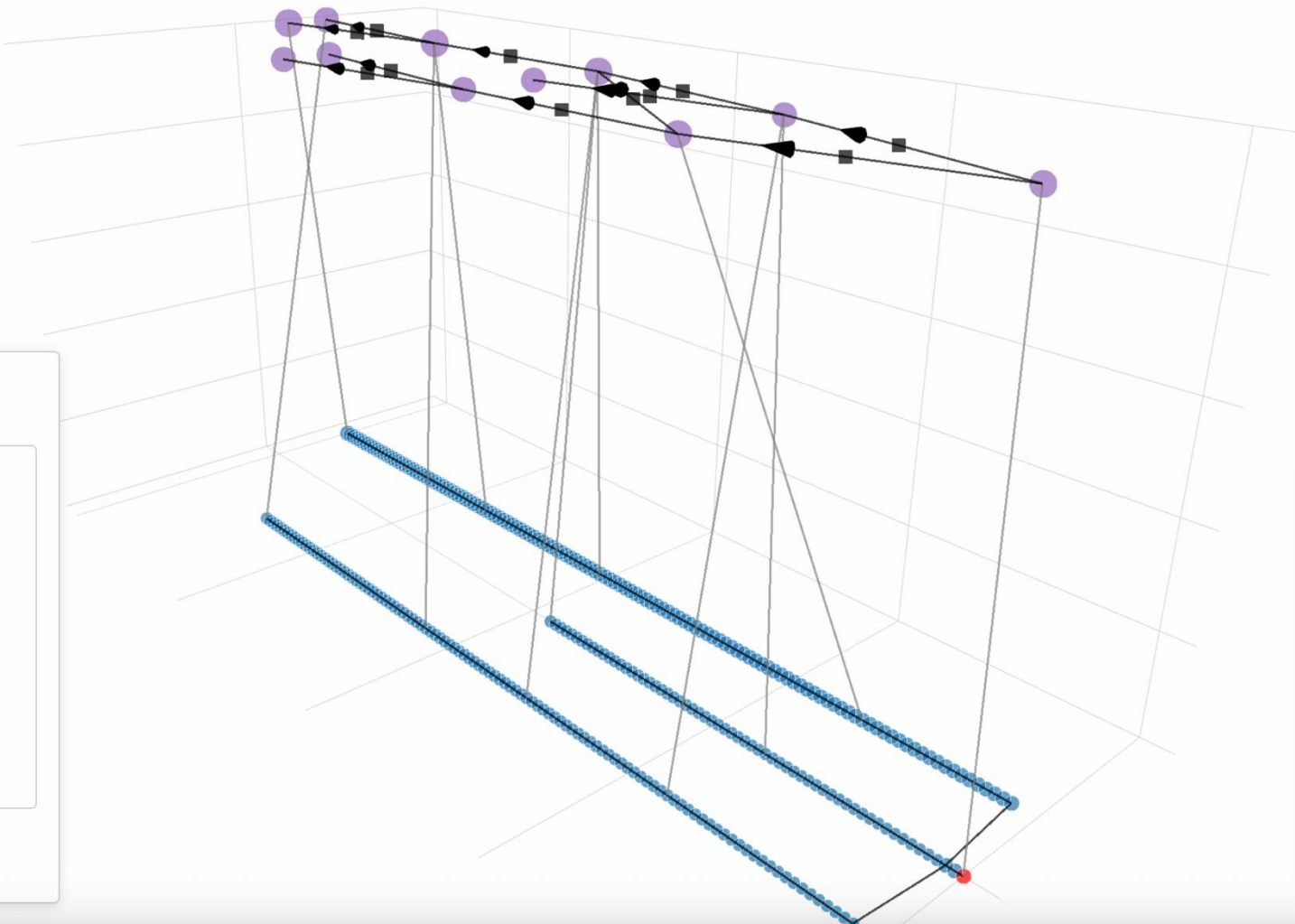
Pick up pot

Pour into cup



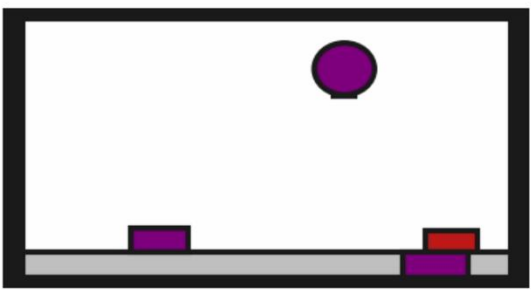
- Concrete states
- Abstract states
- Abstraction

One Remedy: Try Multiple Abstract Plans



Selected Node: x:0

Visualization:



Timeline: 1 / 297

Speed (ms):

Red = current; darker = visited.

hold prev/next to auto-step.

Zoom

Buttons zoom past the scroll-wheel limit.

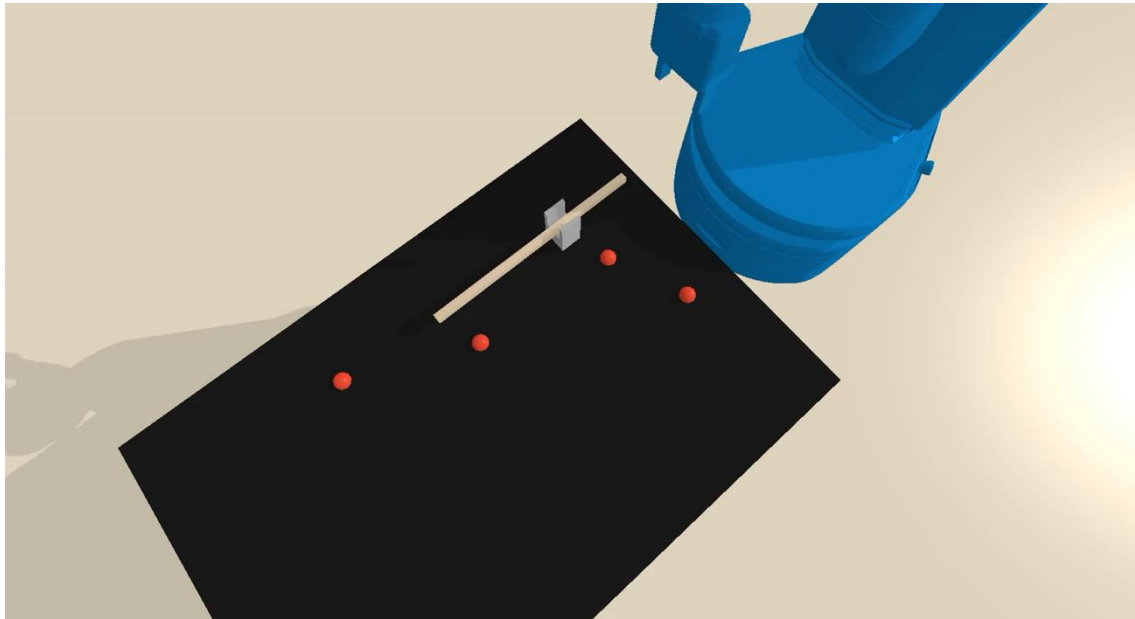
Nodes: 308

Let's Look at the Code

https://github.com/Princeton-Robot-Planning-and-Learning/prpl-mono/blob/main/bilevel-planning/src/bilevel_planning/bilevel_planners/sesame_planner.py

Let's Be Clear About How Terrible Naively Generating Abstract Plans Can Be

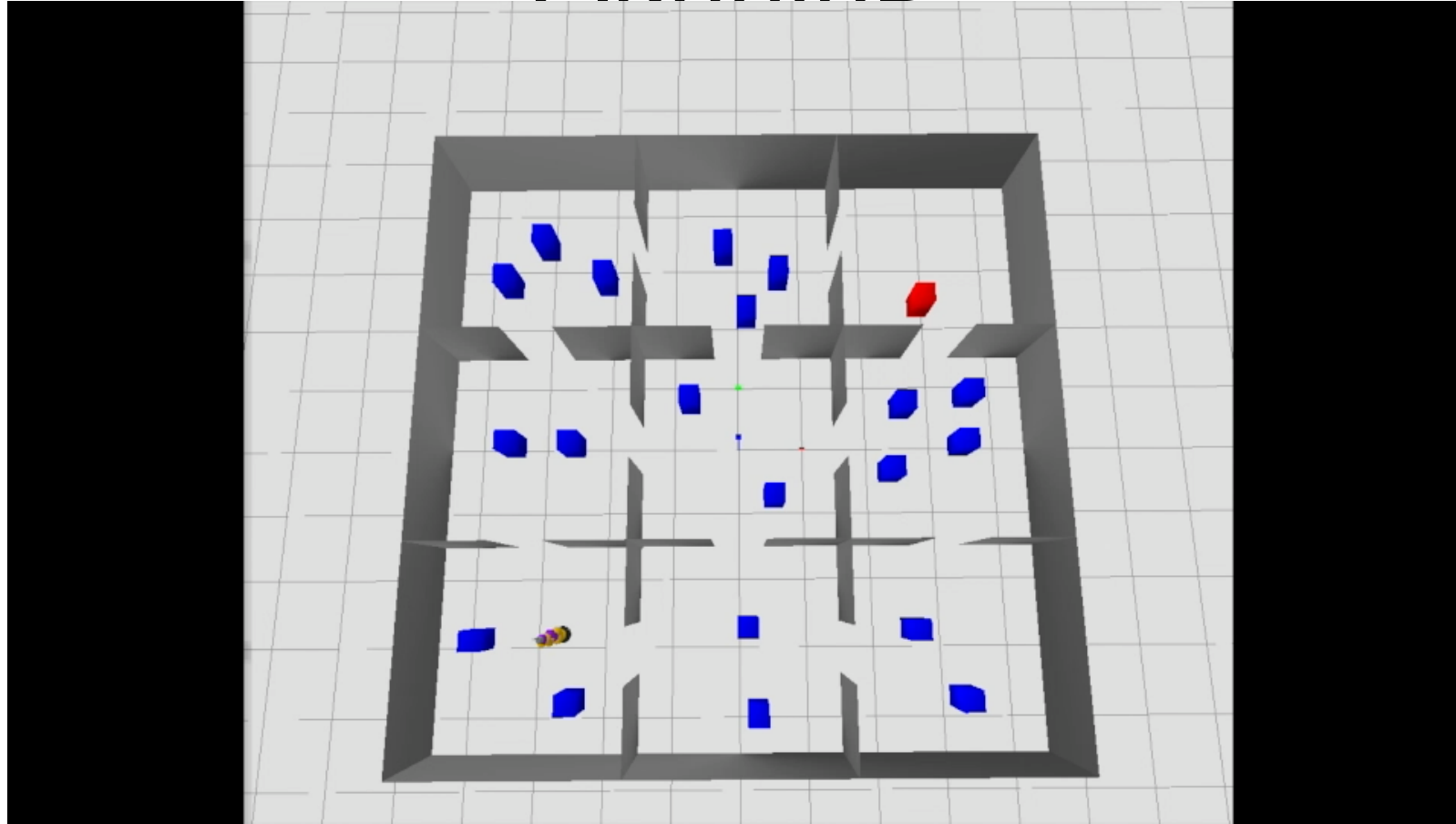
`max_abstract_plans`: the maximum number of abstract plans to try before giving up



```
Poke(button1)  
Poke(button2)  
Poke(button3)  
Poke(button4)
```

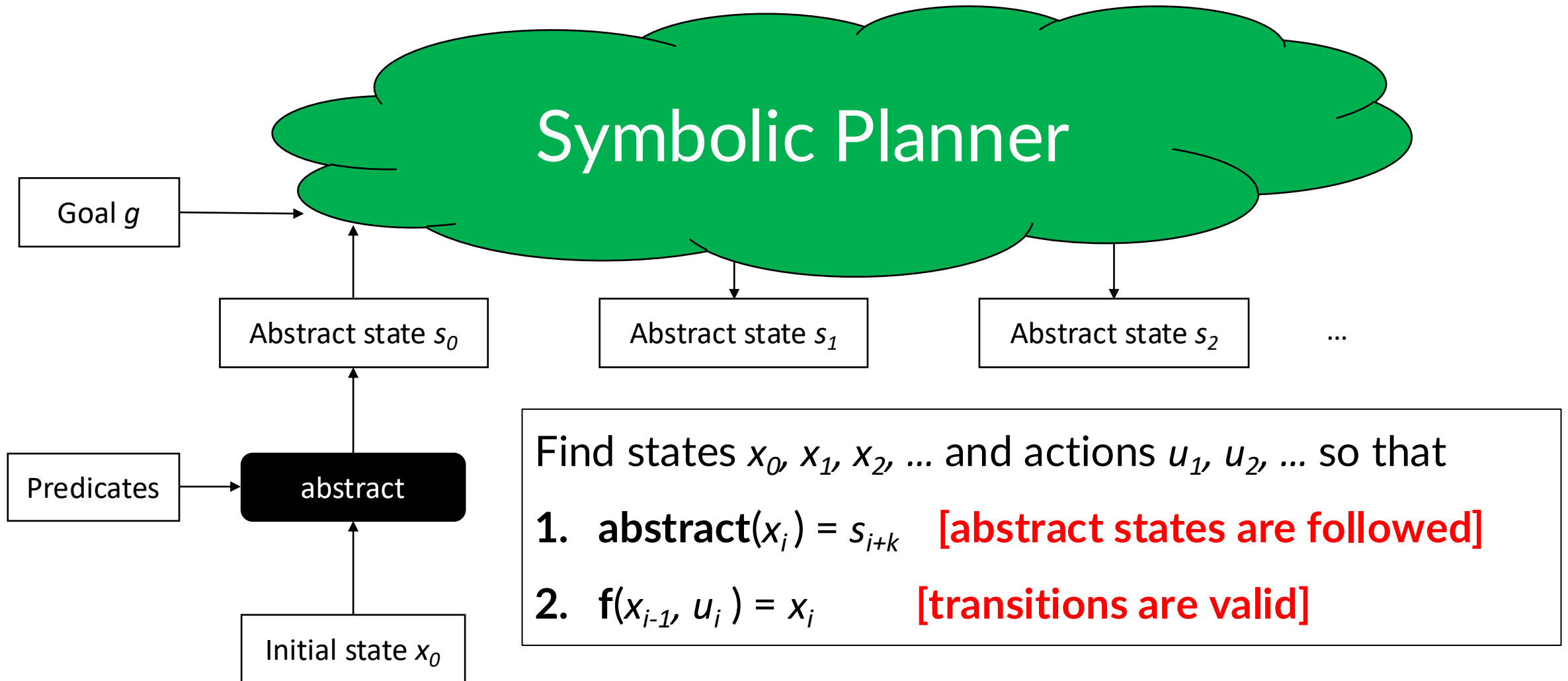
Well that didn't work...

Using Failures to Inform Abstract Planning



See, e.g., “Navigation Among Movable Obstacles” Mike Stilman 2007.

Bilevel Planning: View Abstractions as *Constraints*



Review: Important Parameters

`max_abstract_plans`: the maximum number of abstract plans to try before giving up

`num_sampling_attempts_per_step`: the number of samples to draw at each step before backtracking

Morning Agenda

- From Task Planning to Bilevel Planning
- **Introduction to Motion Planning**
- Multi-Modal Motion Planning and Beyond
- TAMP in the Real World (TipTop)

How Should We Choose Abstractions?

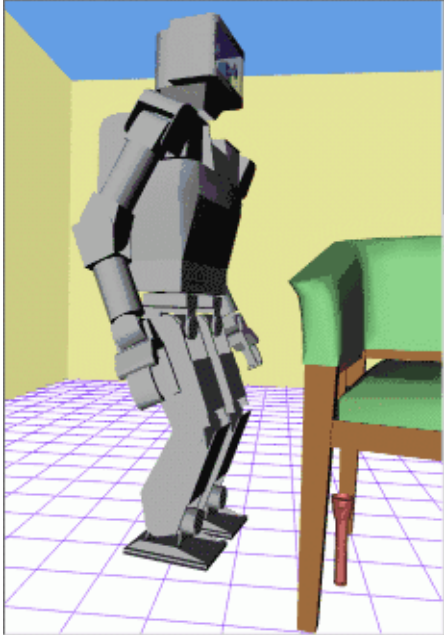
To start, let's focus on **motion in free space**

“We have a brain for one reason and one reason only — and that's to produce adaptable and complex movements.”

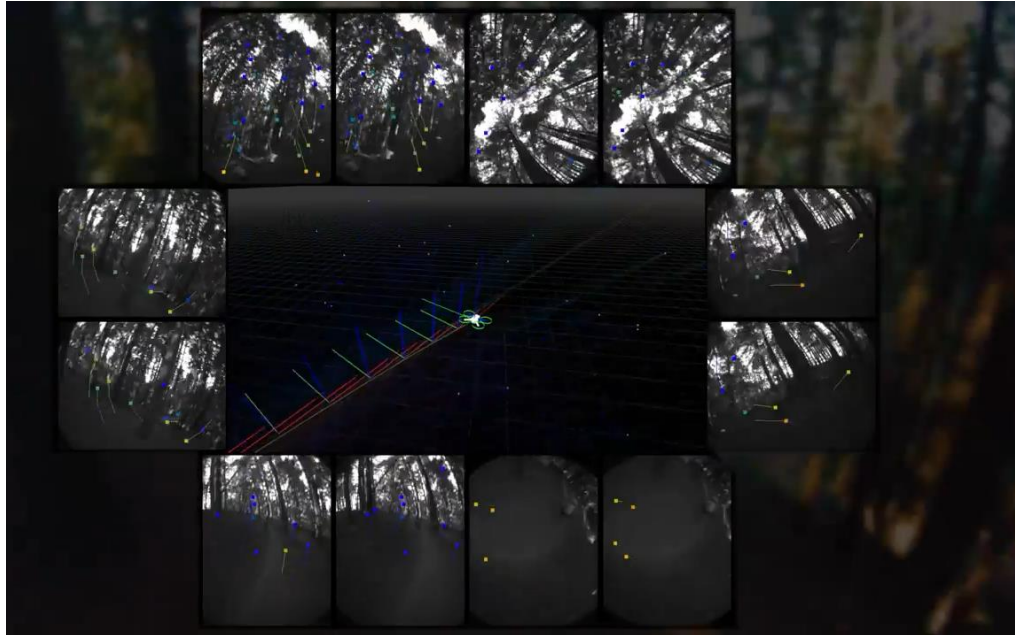
Daniel Wolpert
TED Talk, 2011



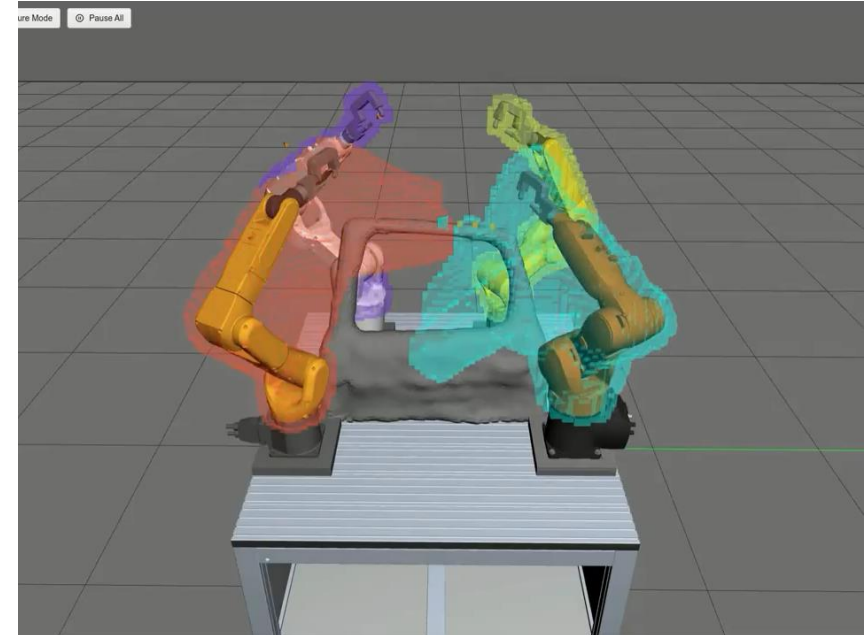
Motion Planning



Kuffner (2002)



Skydio (2019)



Realtime Robotics (2023)

Motion Planning Problem Setting

A motion planning problem includes:

- A *configuration space* $\mathcal{X}$

Must be bounded & have distance metric & have other nice properties...

- An initial configuration $x_0 \in \mathcal{X}$

- A goal configuration $x_g \in \mathcal{X}$

Alternative definition: set of configurations

- A *feasibility check* $f: \mathcal{X} \rightarrow \{T, F\}$

Usually: feasible (T) if robot not in collision

Motion Planning Solution

A *solution* to a motion planning problem is a trajectory

$$\alpha: [0, H] \rightarrow \mathcal{X}$$

where

1. $\alpha(0) = x_0$
2. $\alpha(H) = x_g$
3. The robot can follow the trajectory

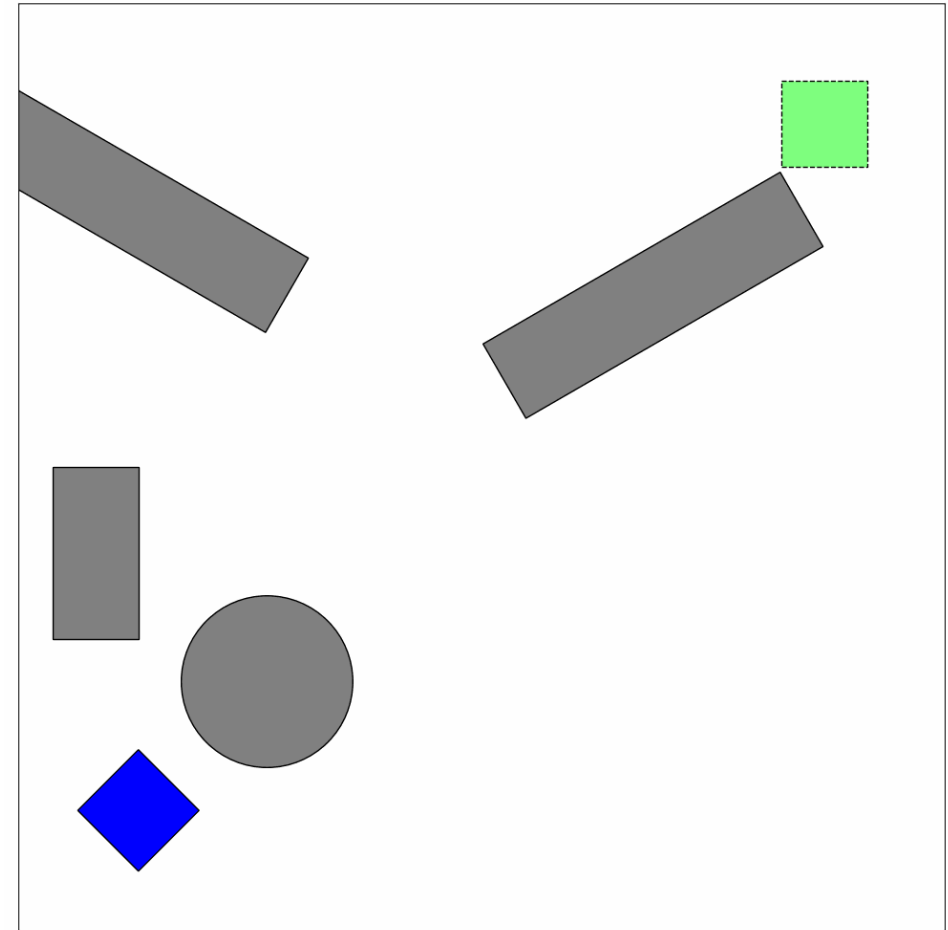
Continuous states *and* continuous time!

Many different ways to specify this

Actions are implicit

Example: Motion Planning with 2D Shapes

- Configuration space:
 - (x position, y position, rotation)
 - x and y position are bounded
 - Subset of $SE(2)$
- Initial configuration: see image
- Goal configuration: see image
- Feasibility check:
 - Configuration is feasible if robot is not in collision with obstacles



A Stupidest Possible Algorithm

1. Discretize the configuration space
2. Run path planning (e.g., A^* with distance-to-goal heuristic)

When will this go badly?

Is this sound, complete, optimal?

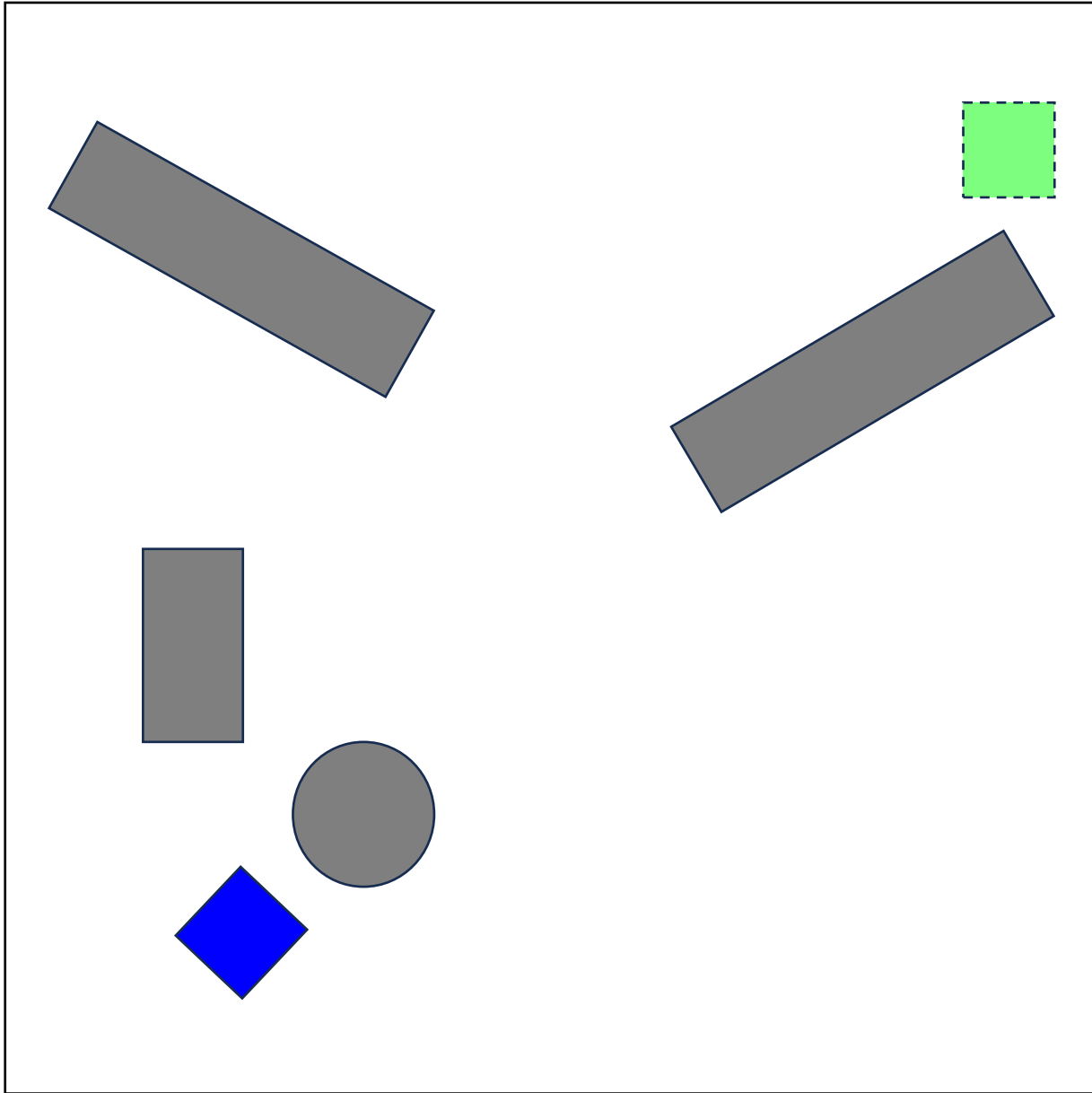
Sampling-Based Motion Planning

A yellow speech bubble with a tail pointing to the left, containing the text "Let's assume we can sample from the configuration space".

Let's assume we can sample from the configuration space

Rapidly Exploring Random Trees (RRT)

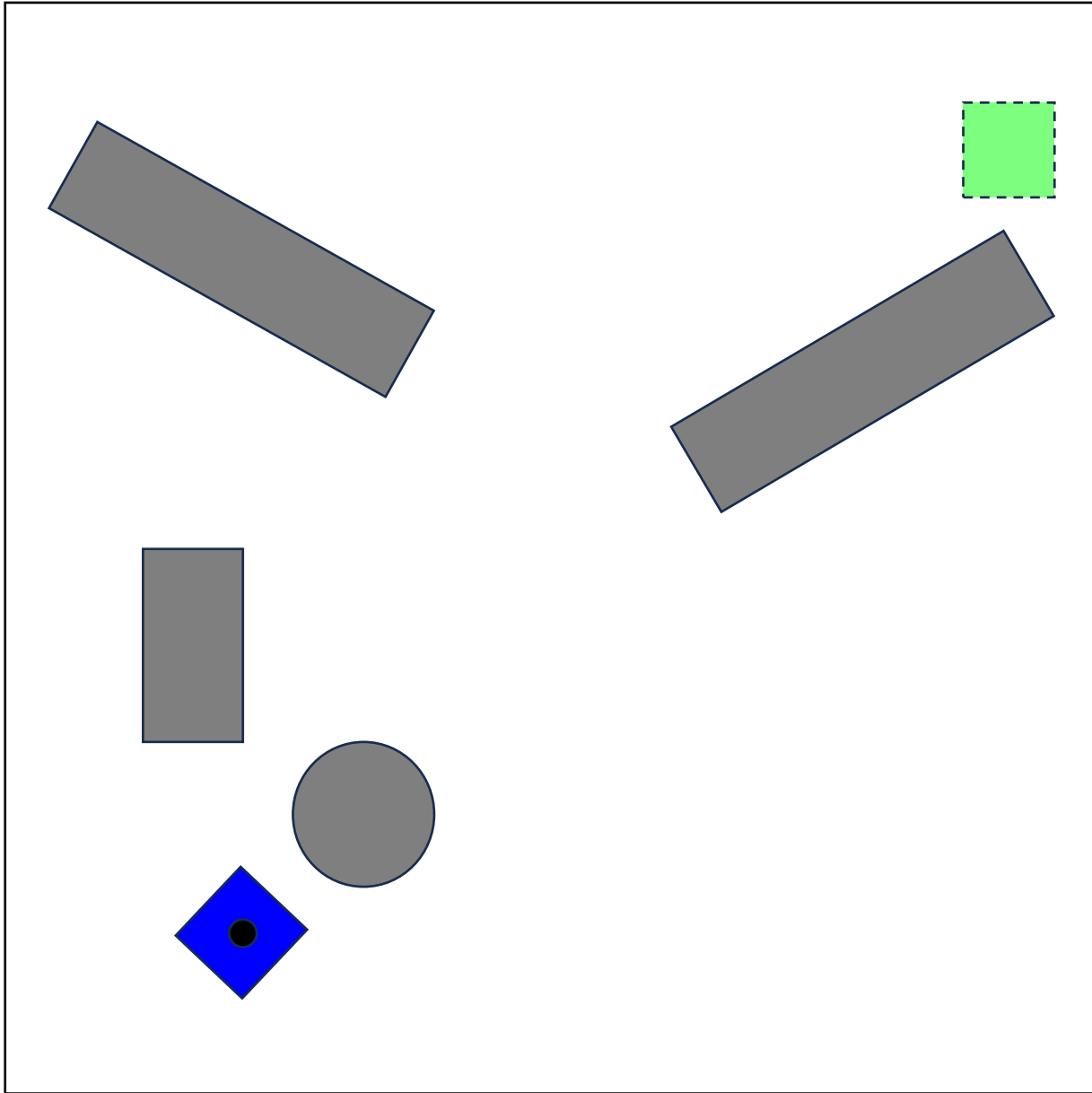
```
RRT( $x_0, x_g, \mathcal{X}, f$ )
1  // Initialize tree at  $x_0$ 
2  nodes = [Node( $x_0$ )]
3  repeat:
4      if uniform() < goalSampleProb
5          // Try to go directly to the goal
6           $x_{\text{target}} = x_g$ 
7      else
8          // Sample a target configuration
9           $x_{\text{target}} = \text{sample}(\mathcal{X})$ 
10     // Extend the tree towards the target
11     node = getClosest(nodes,  $x_{\text{target}}$ )
12      $x_{\text{node}} = \text{node.conf}$ 
13     for  $x$  in extend( $x_{\text{node}}, x_{\text{target}}$ )
14         if  $f(x)$ :
15             nodes.add(Node( $x$ ))
16             if  $x = x_g$ :
17                 return finish(nodes)
18     else
19         break
```



```

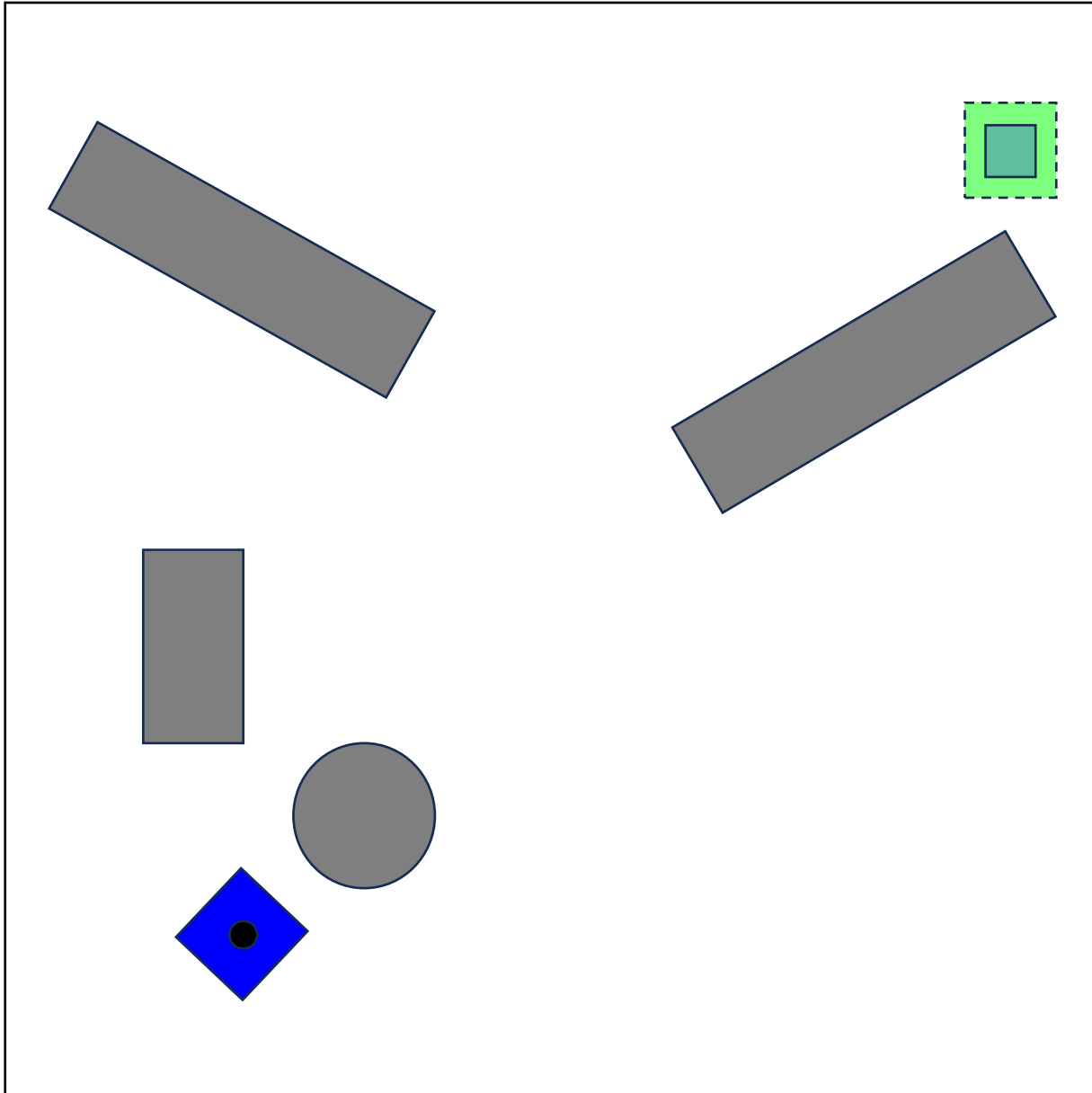
RRT( $x_0, x_g, \mathcal{X}, f$ )
1  // Initialize tree at  $x_0$ 
2  nodes = [Node( $x_0$ )]
3  repeat:
4      if uniform() < goalSampleProb
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8          // Sample a target configuration
9           $x_{\text{target}} = \text{sample}(\mathcal{X})$ 
10     // Extend the tree towards the target
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13     for  $x$  in extend( $x_{\text{node}}, x_{\text{target}}$ )
14         if  $f(x)$ :
15             nodes.add(Node( $x$ ))
16             if  $x = x_g$ :
17                 return finish(nodes)
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```



$\text{RRT}(x_0, x_g, \mathcal{X}, f)$

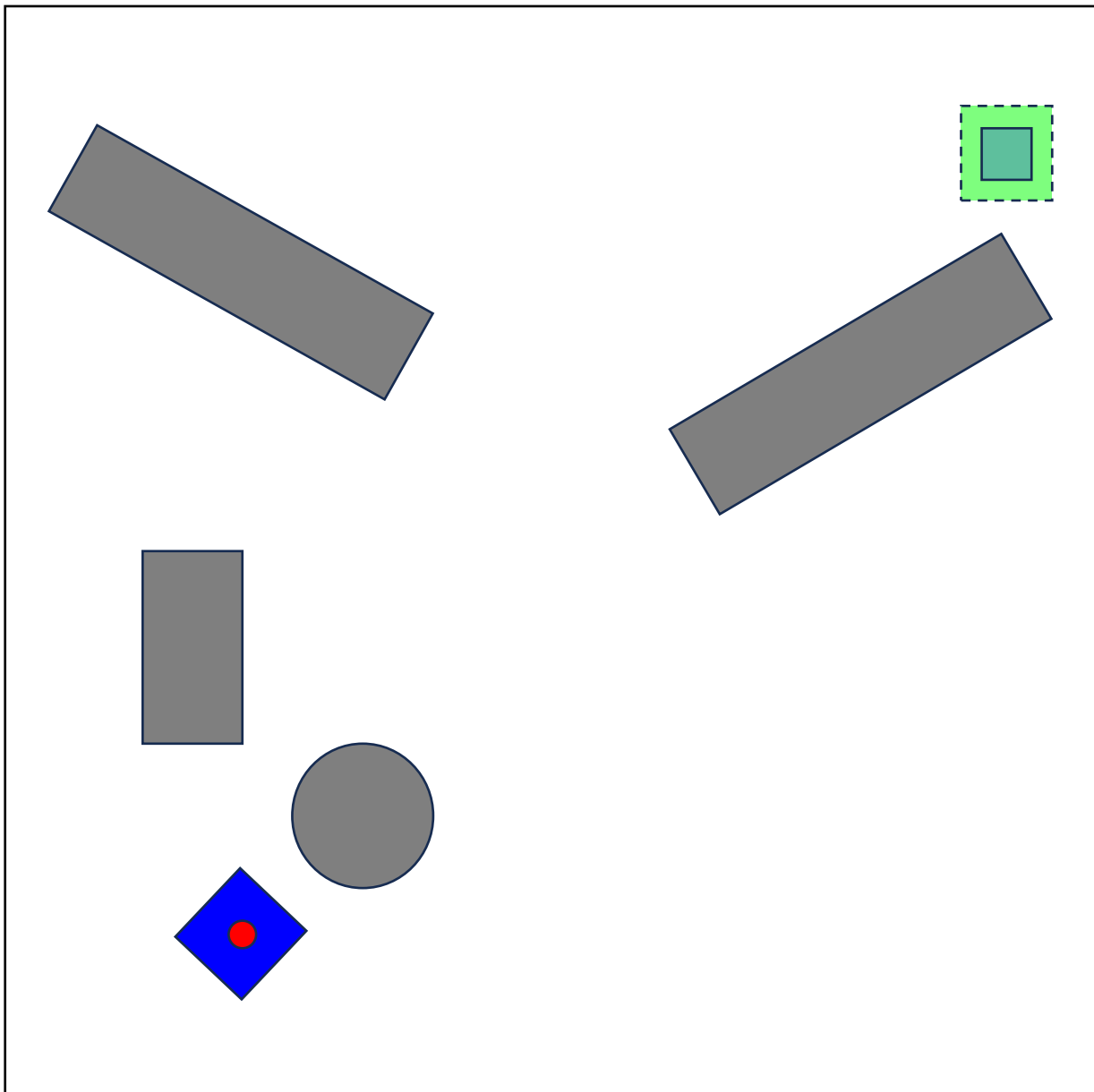
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```

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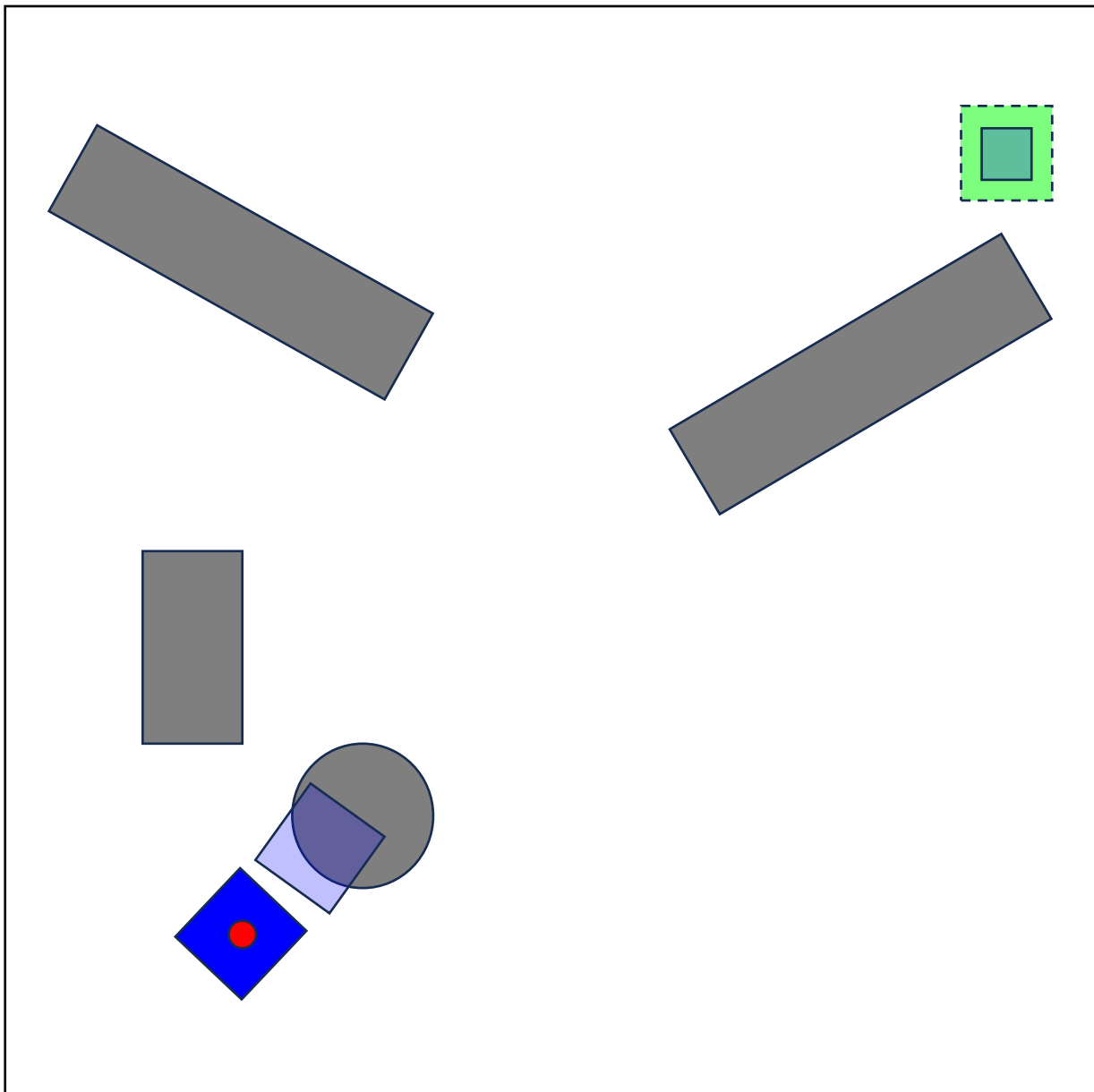
```



```

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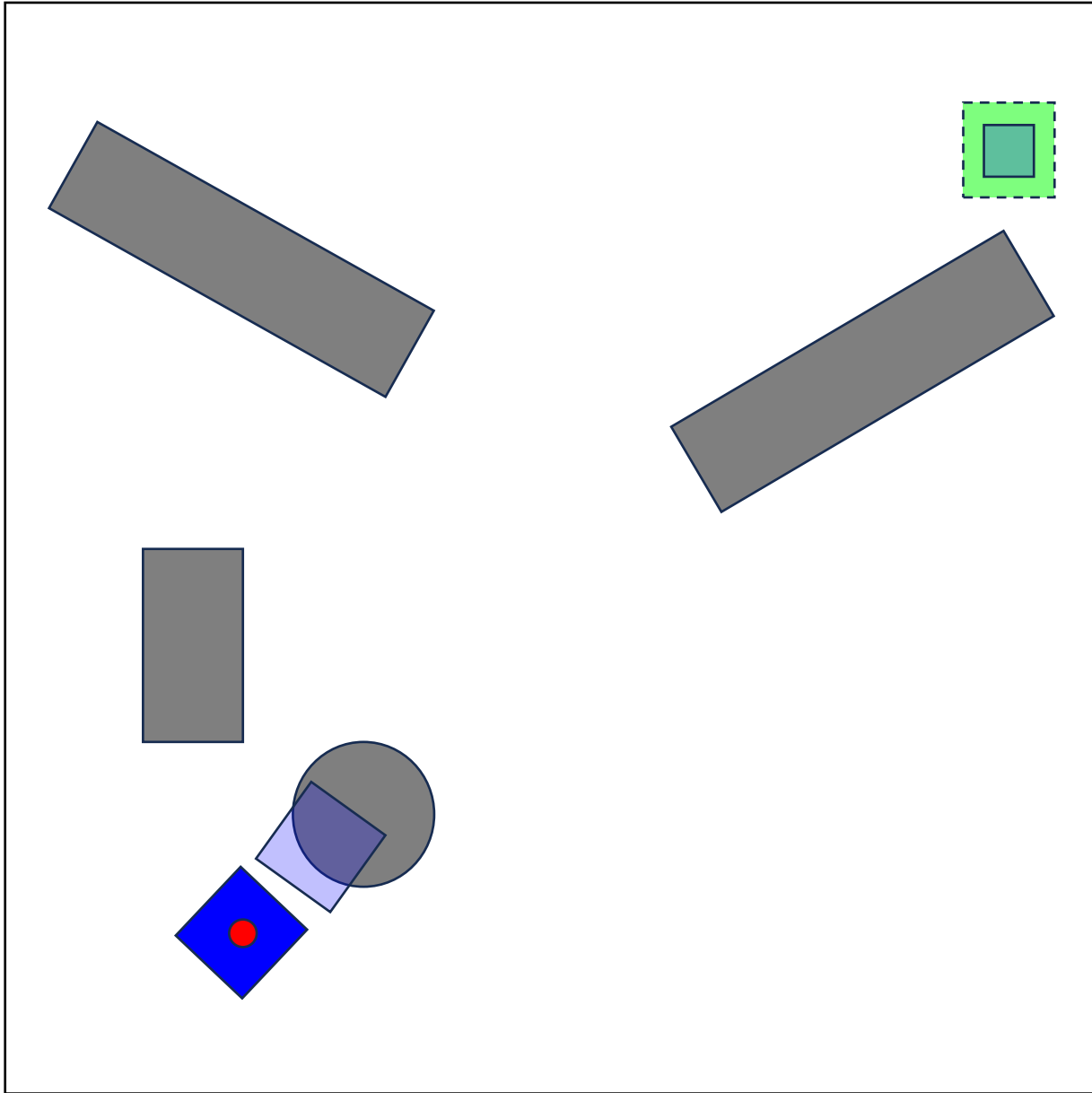
```



```

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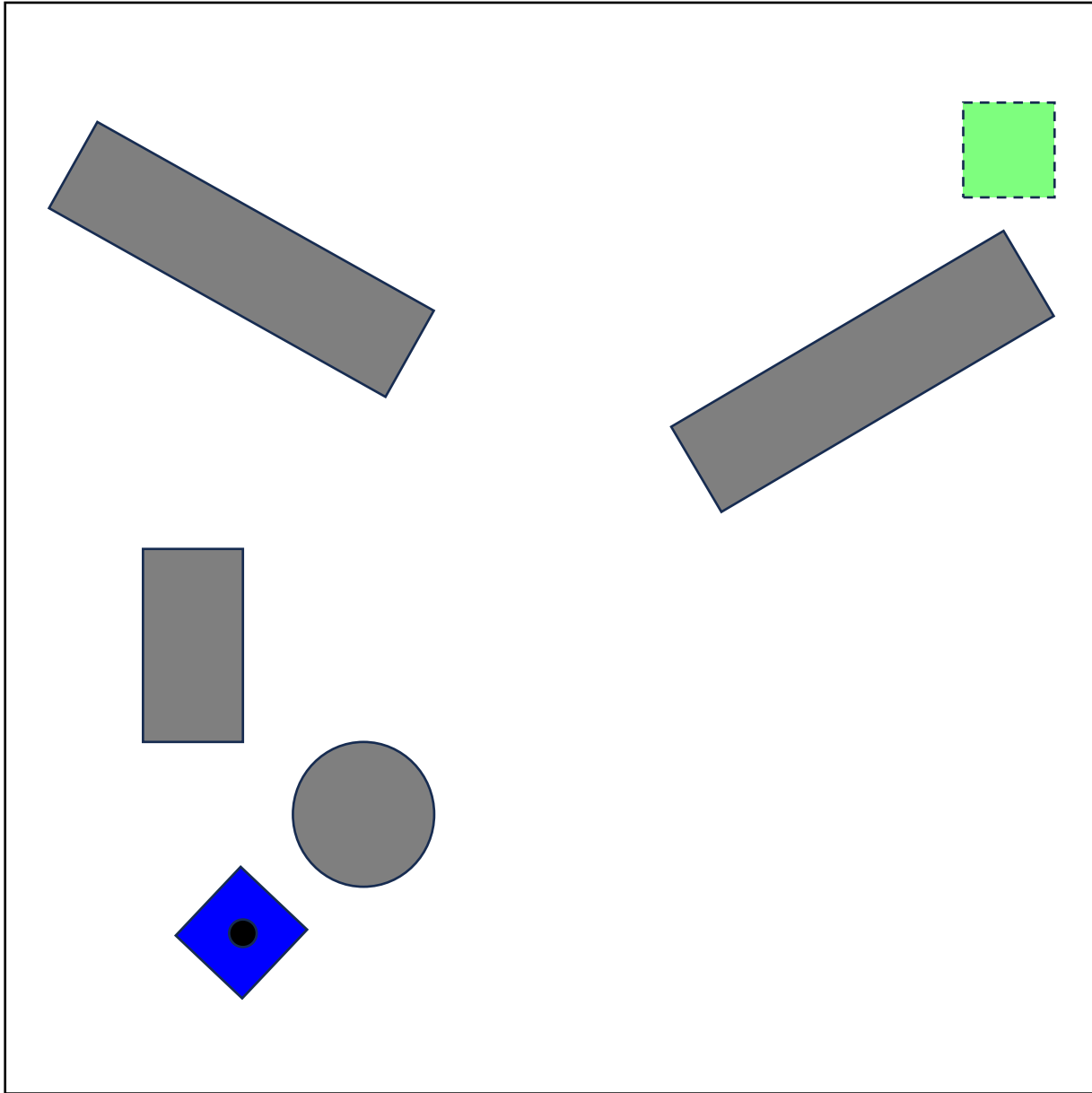
```



```

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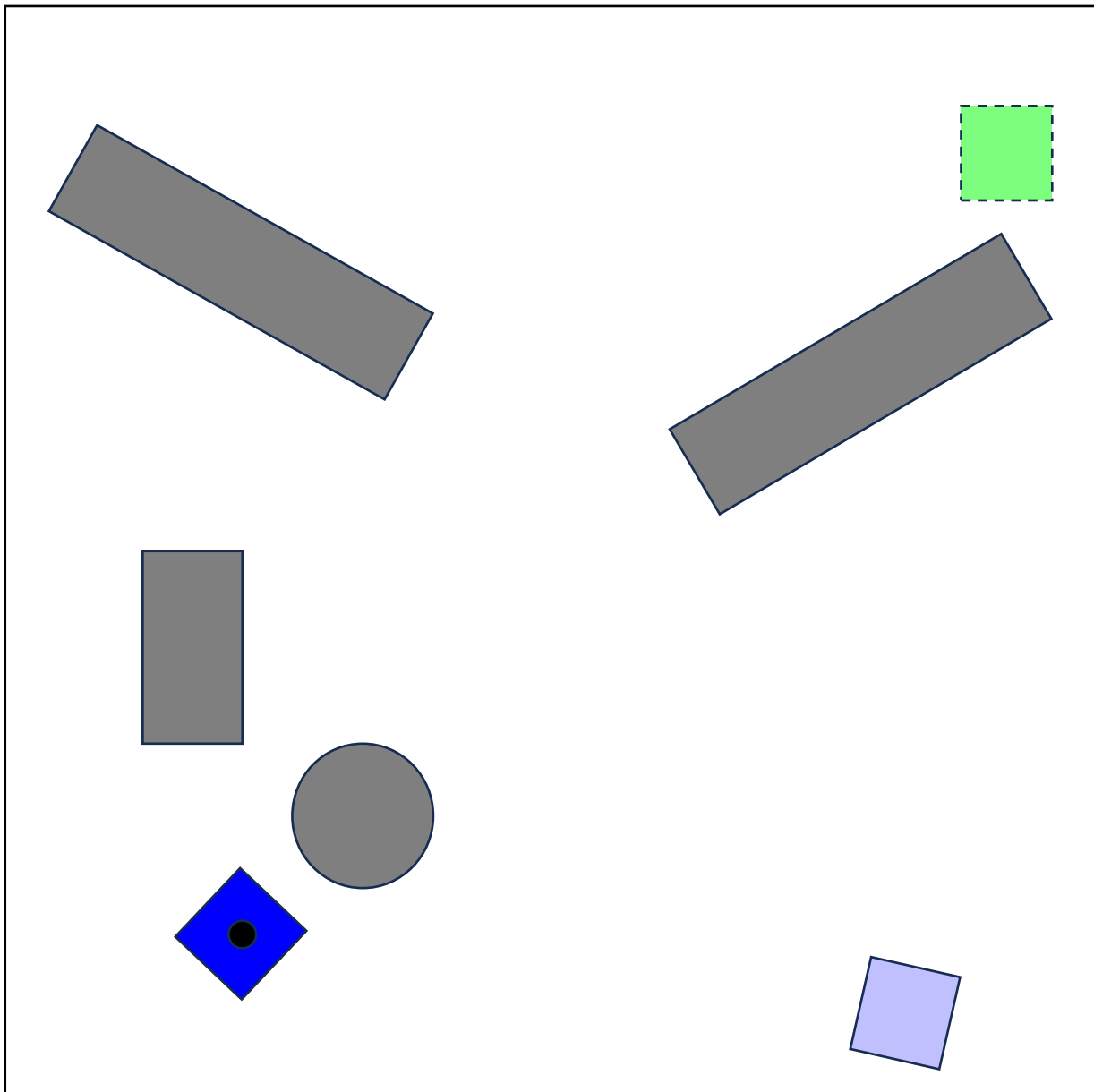
```



```

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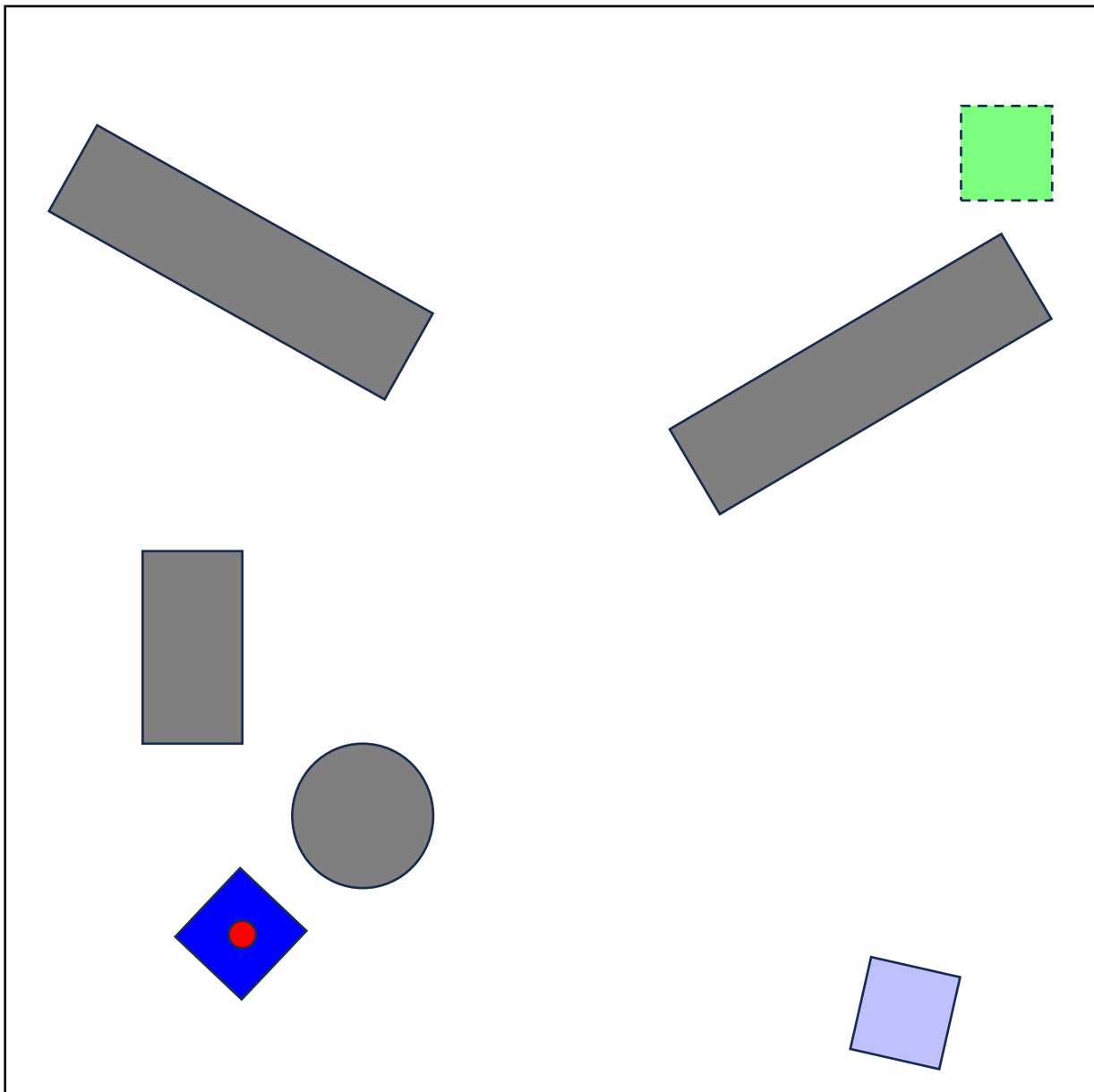
```



```

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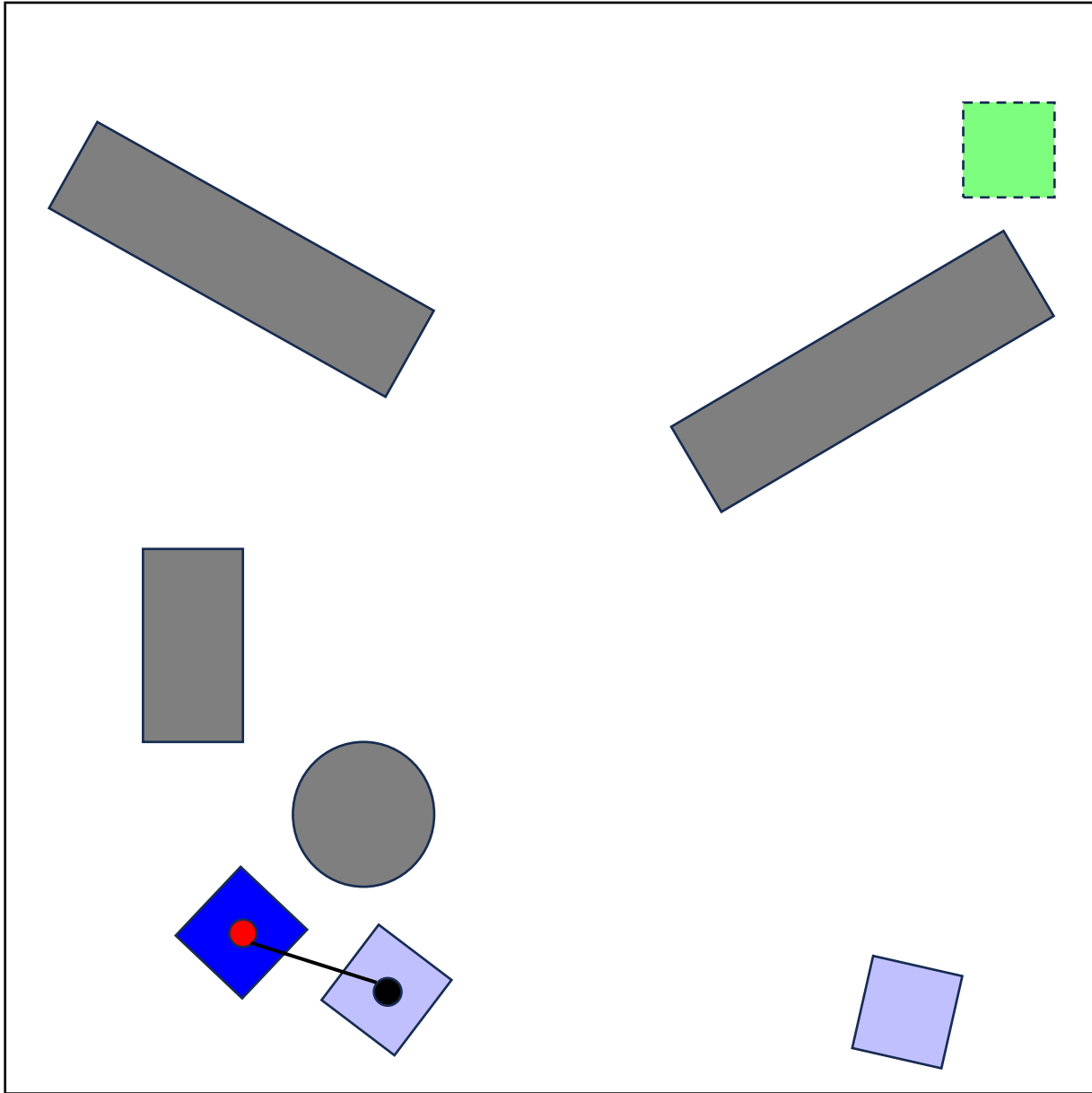
```



```

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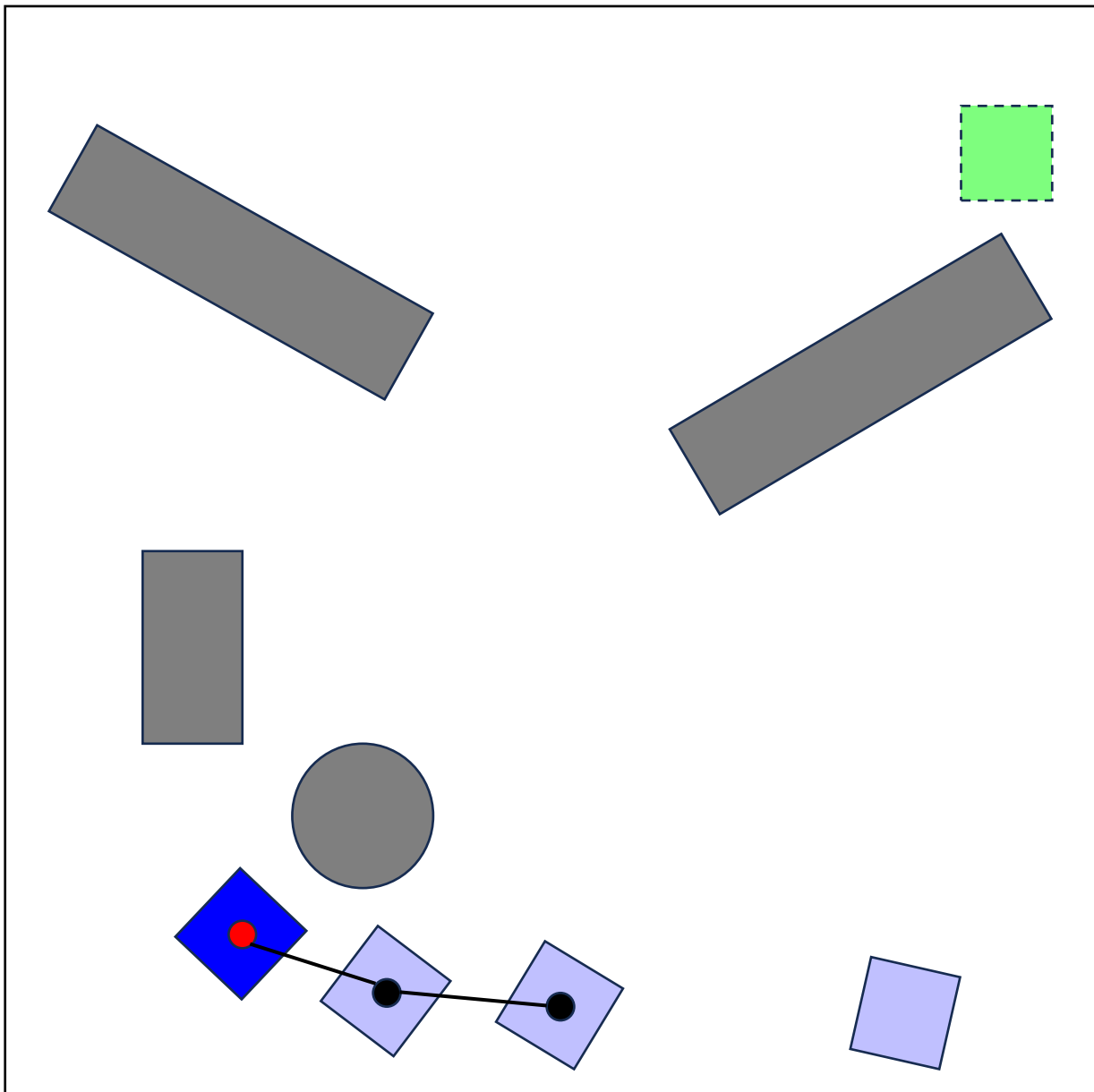
```



```

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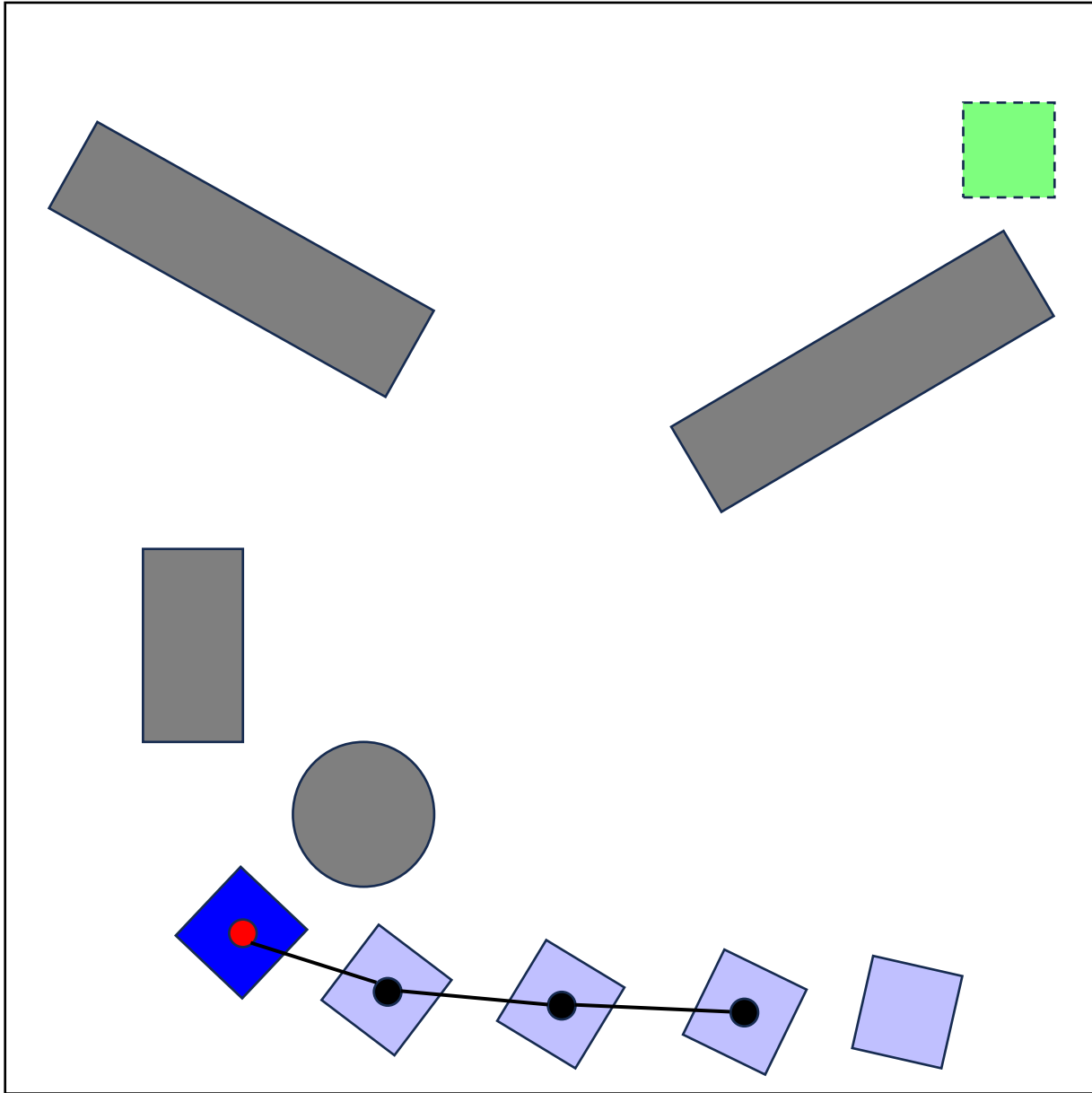
```



```

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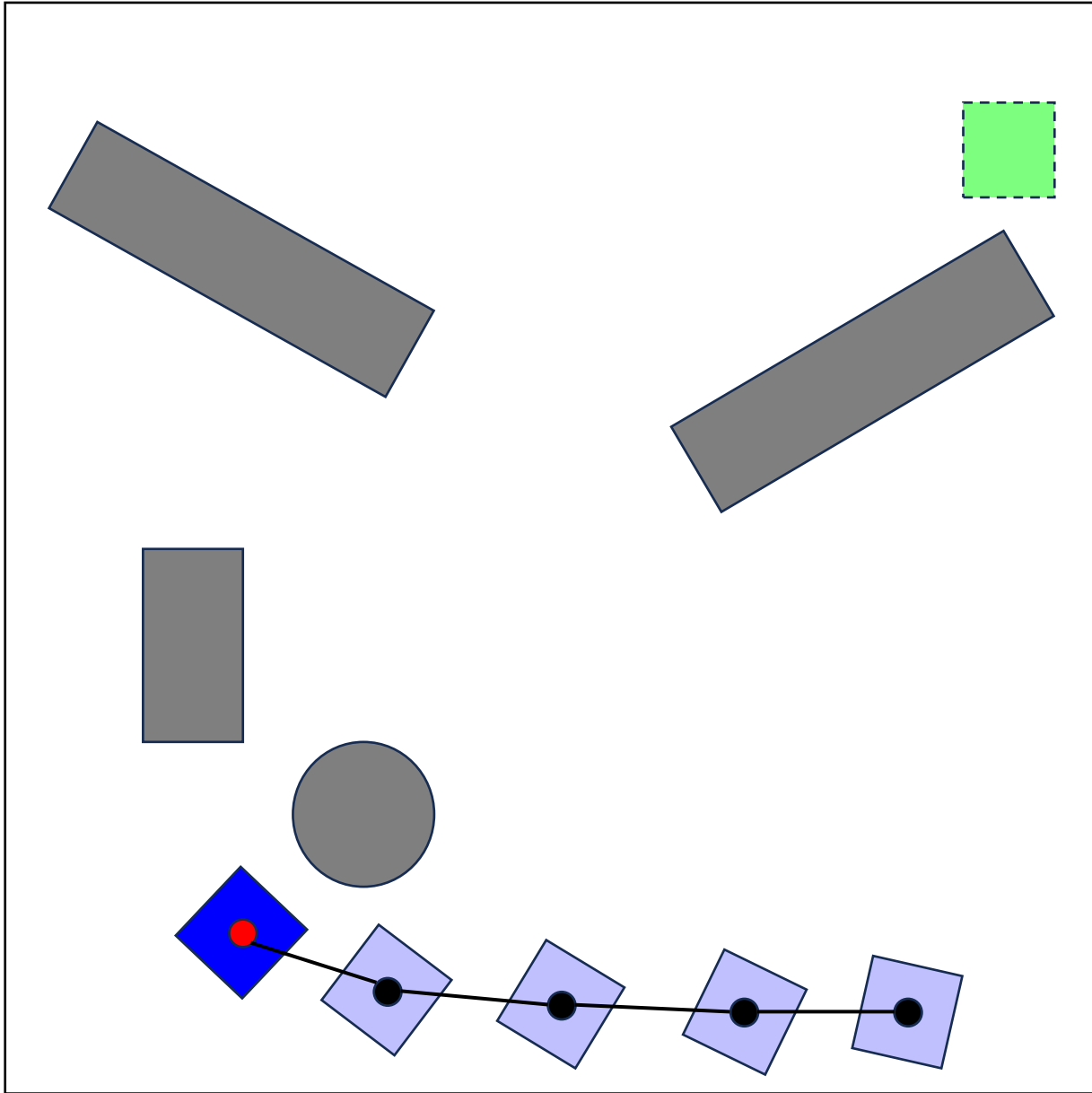


$\text{RRT}(x_0, x_g, \mathcal{X}, f)$

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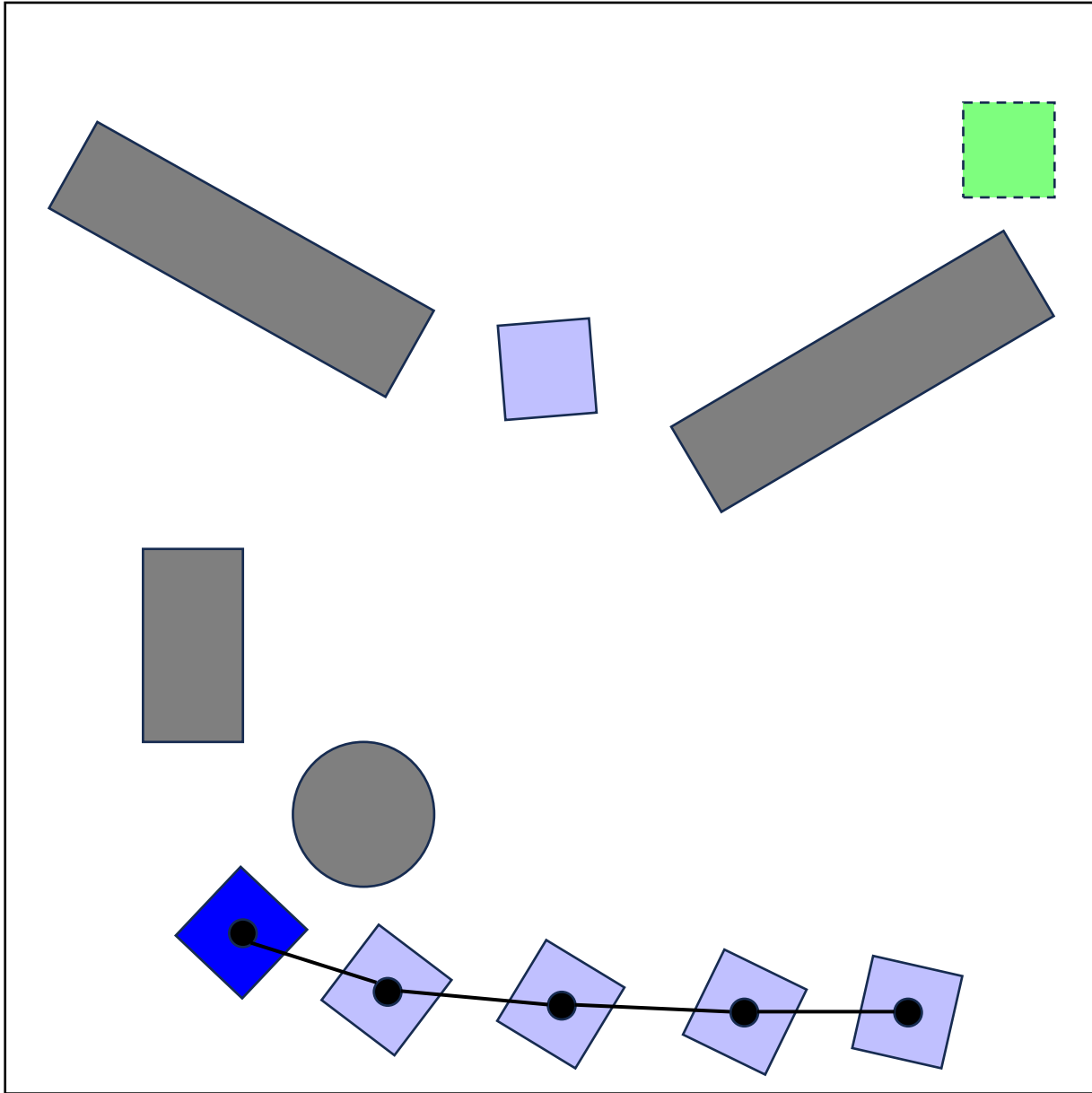


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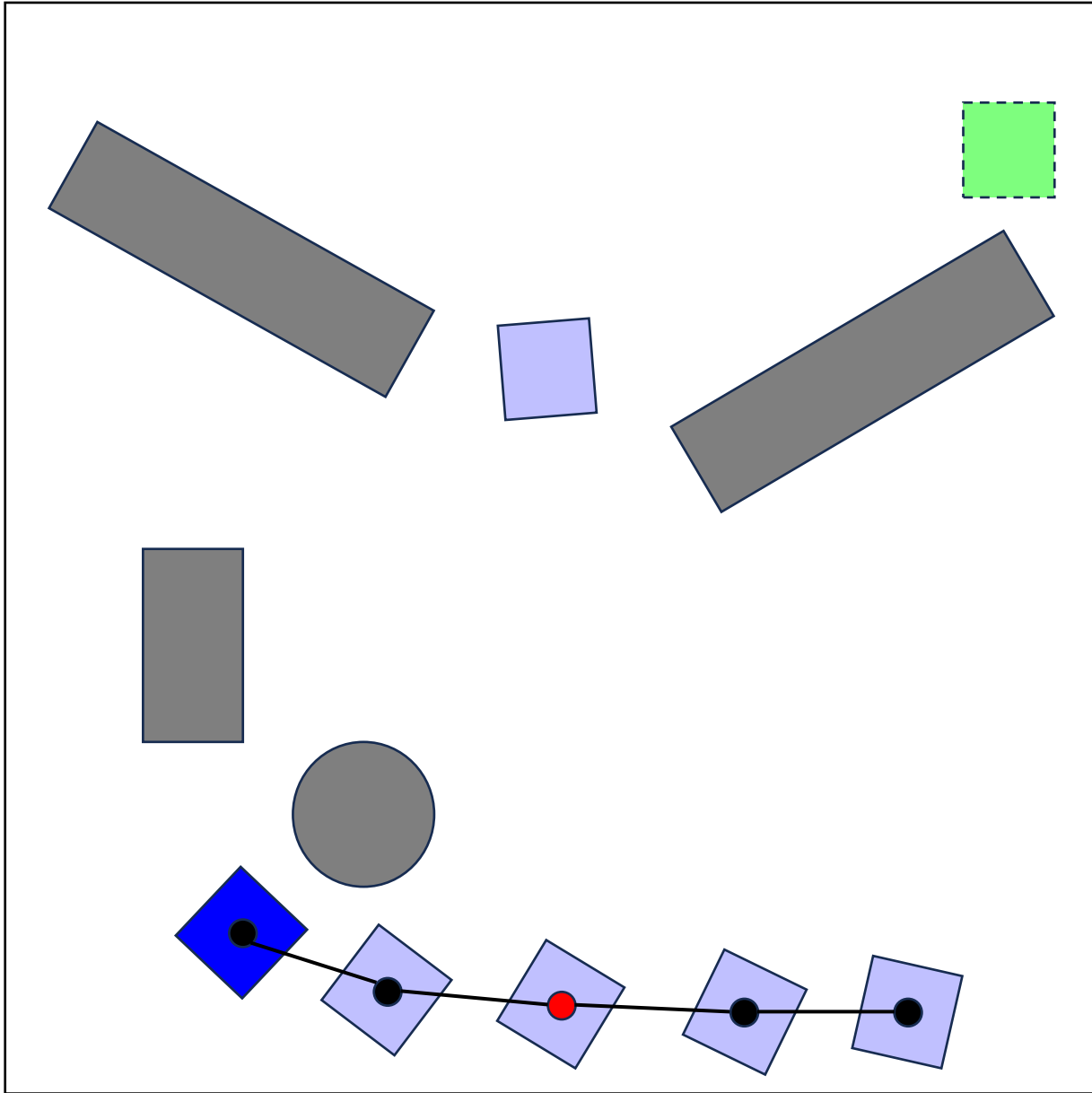


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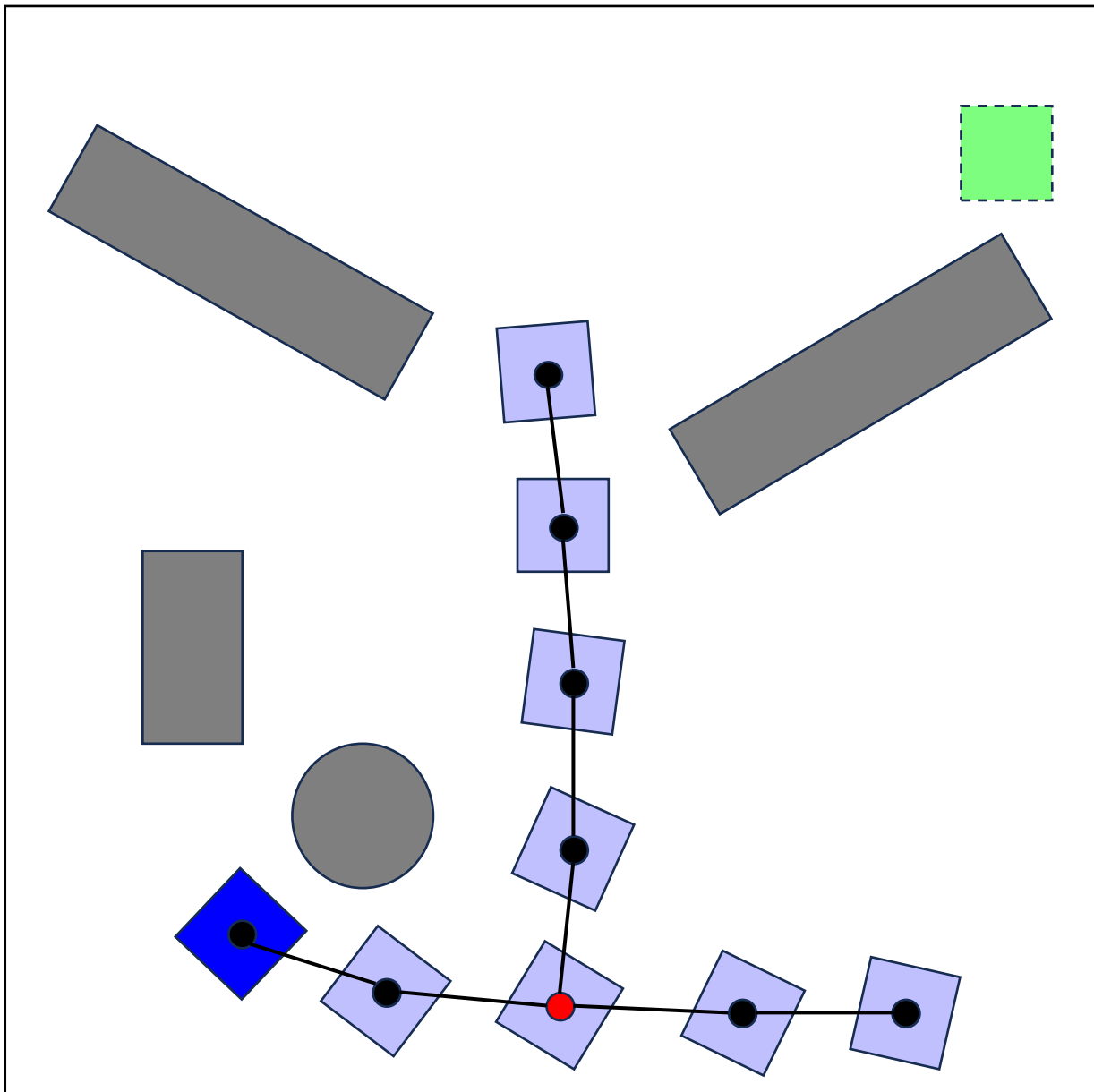


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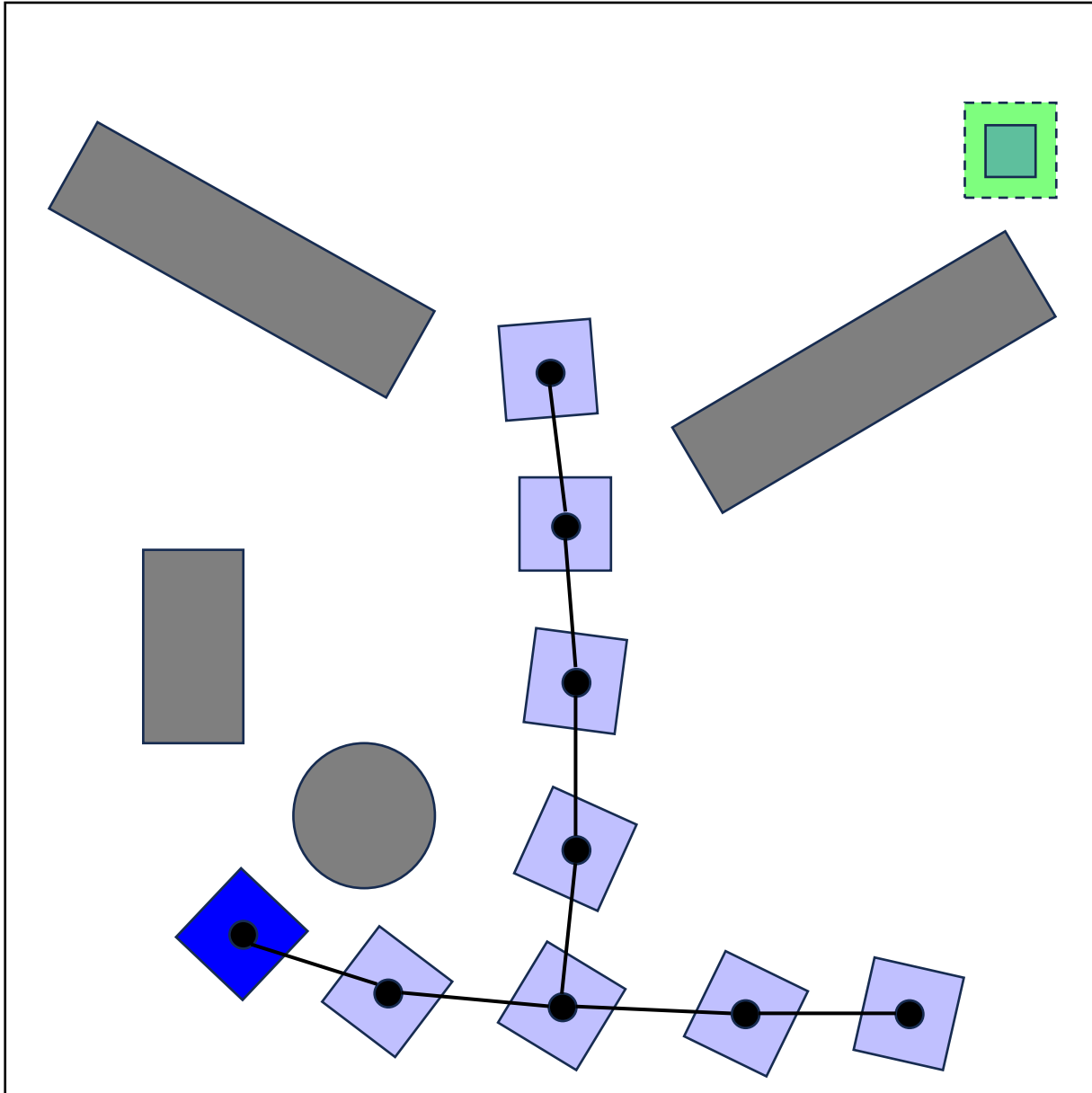
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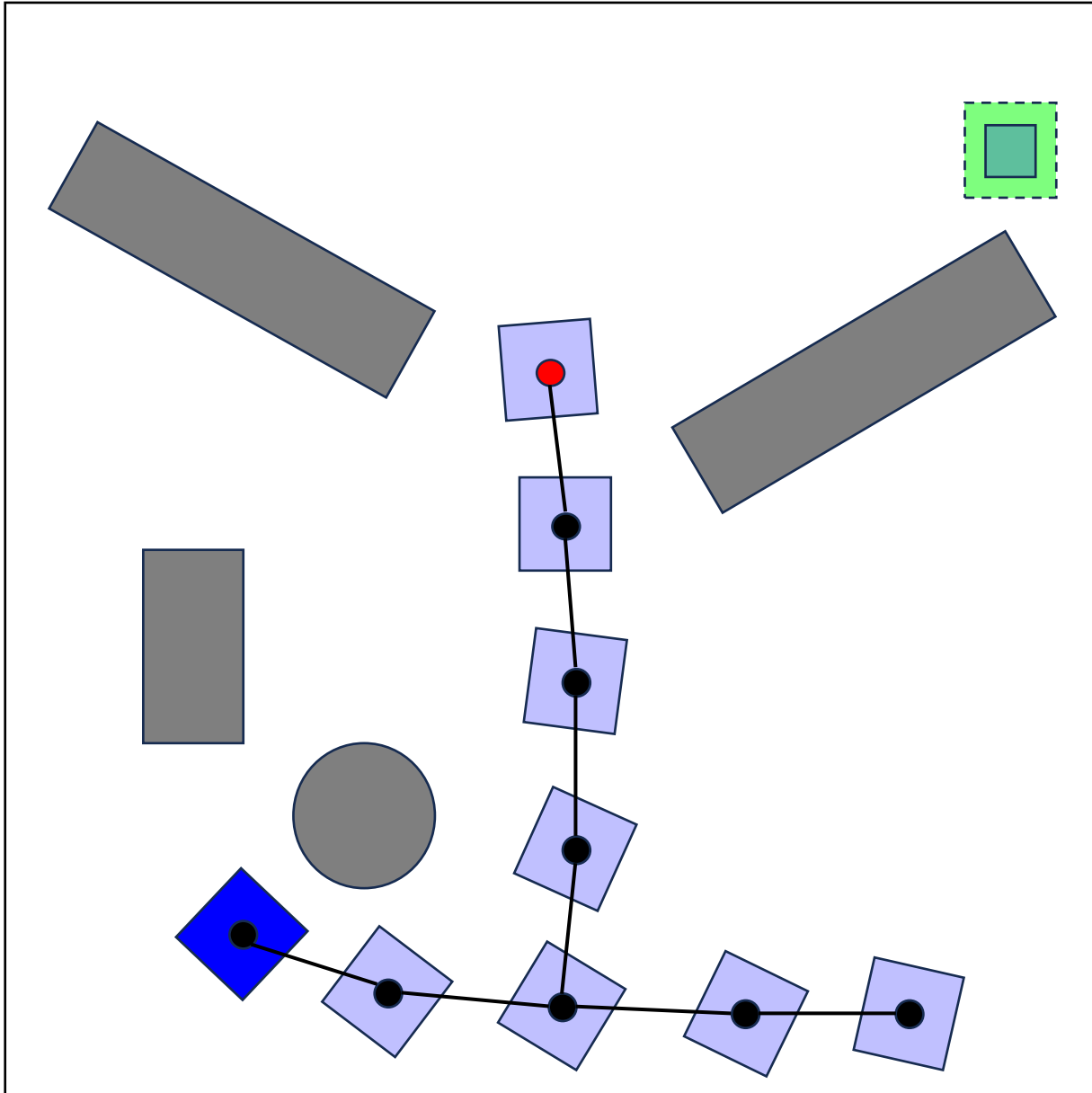


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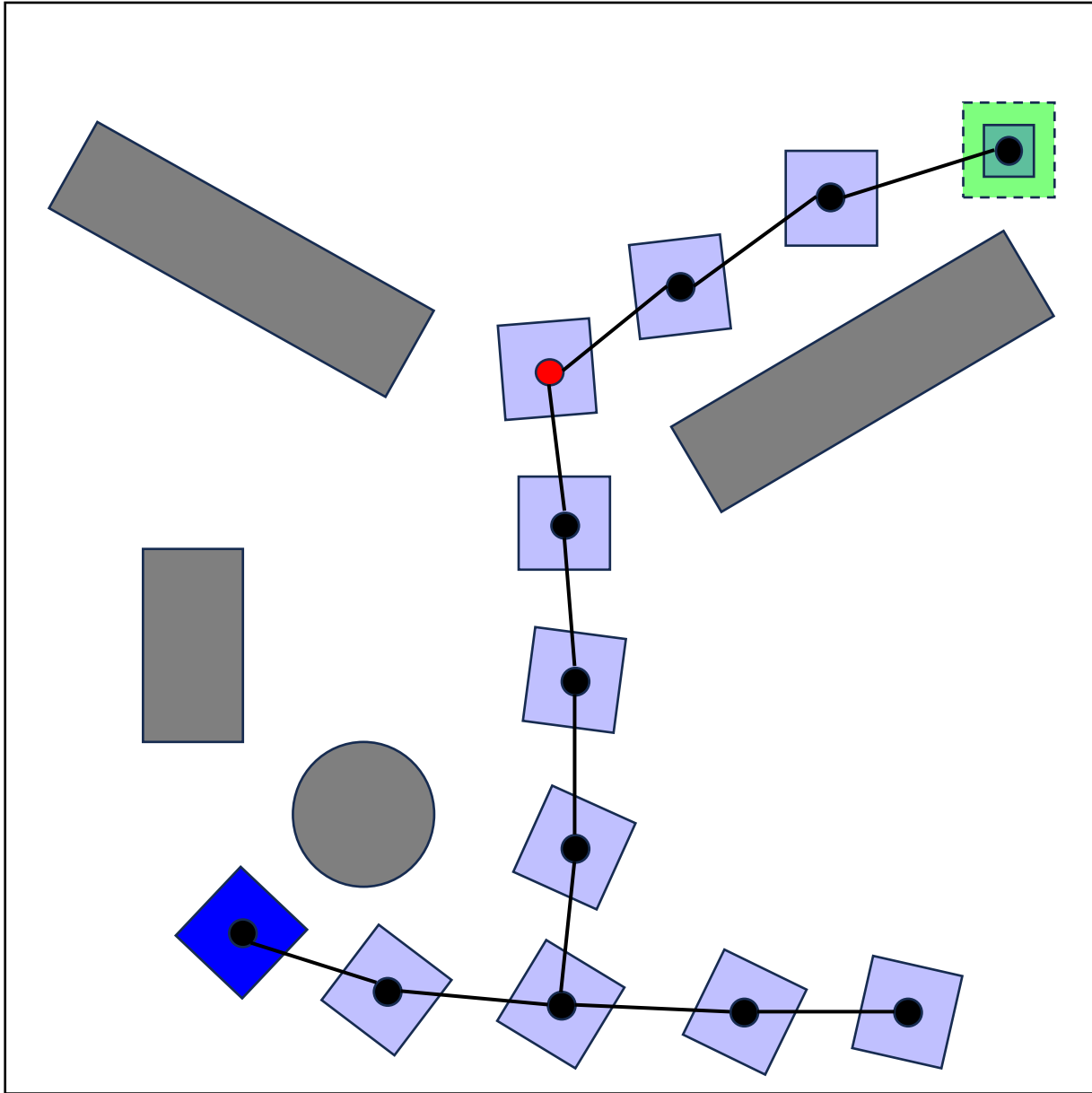
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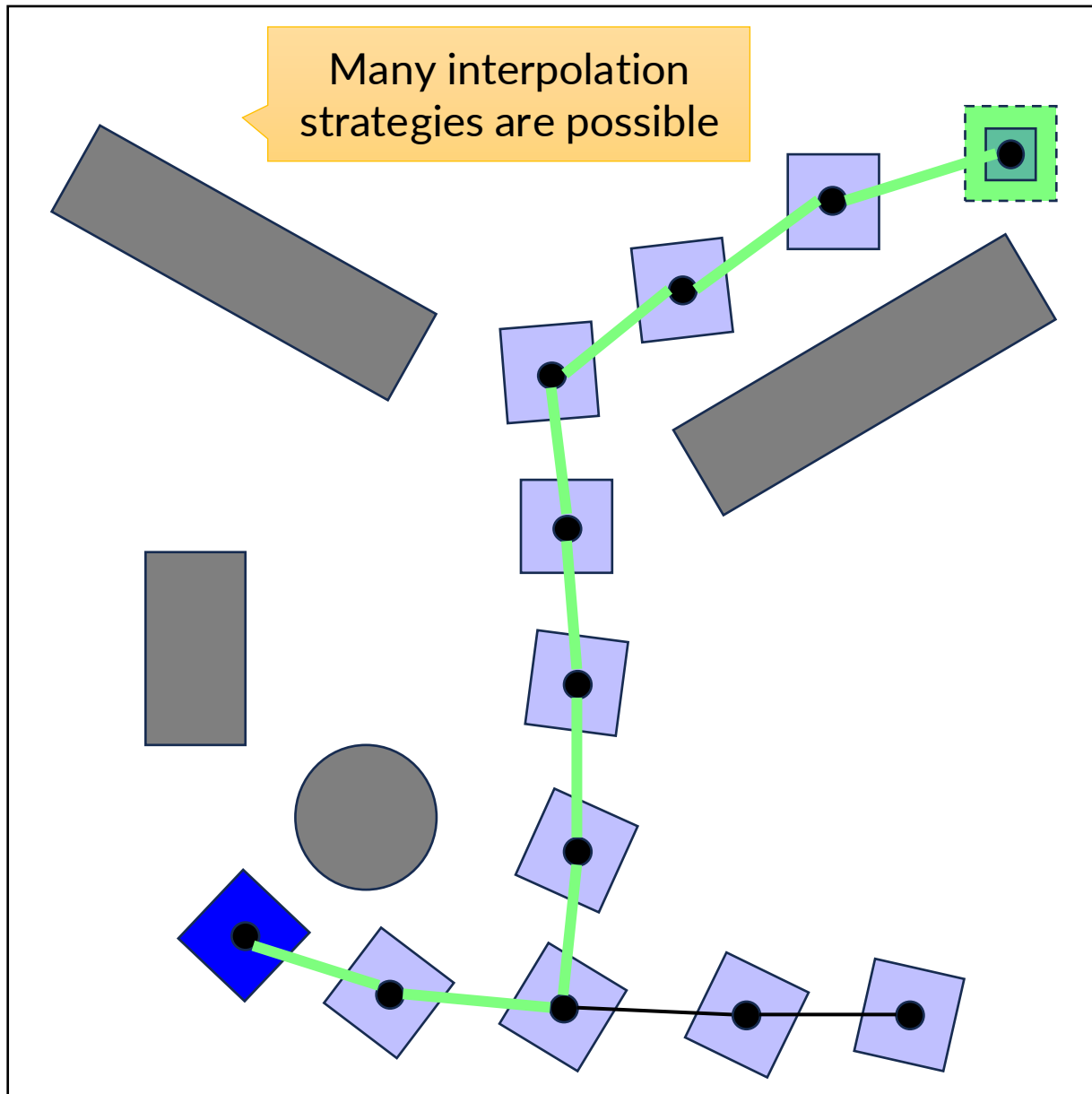


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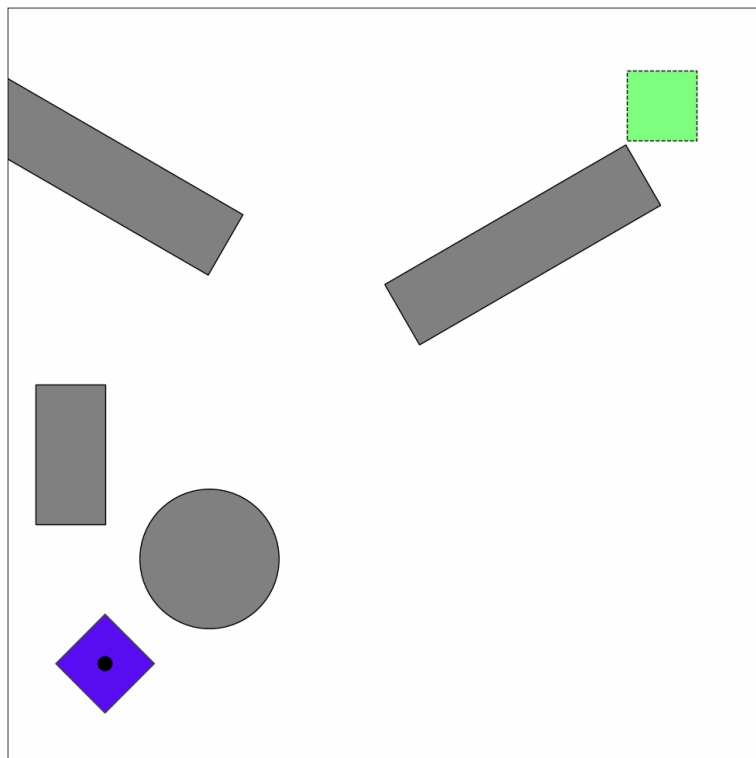
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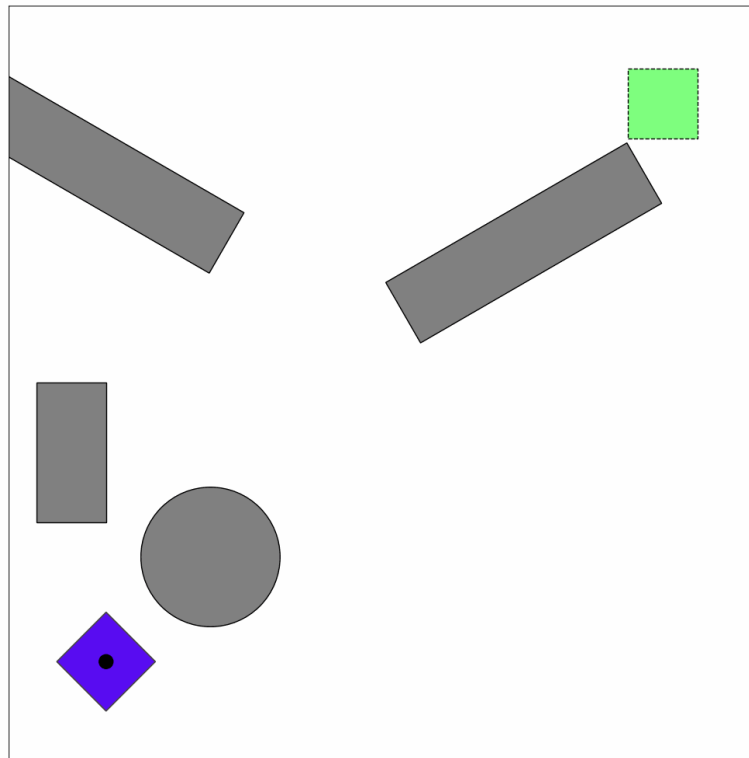
```

Important Hyperparameter: Feasibility Check Distance

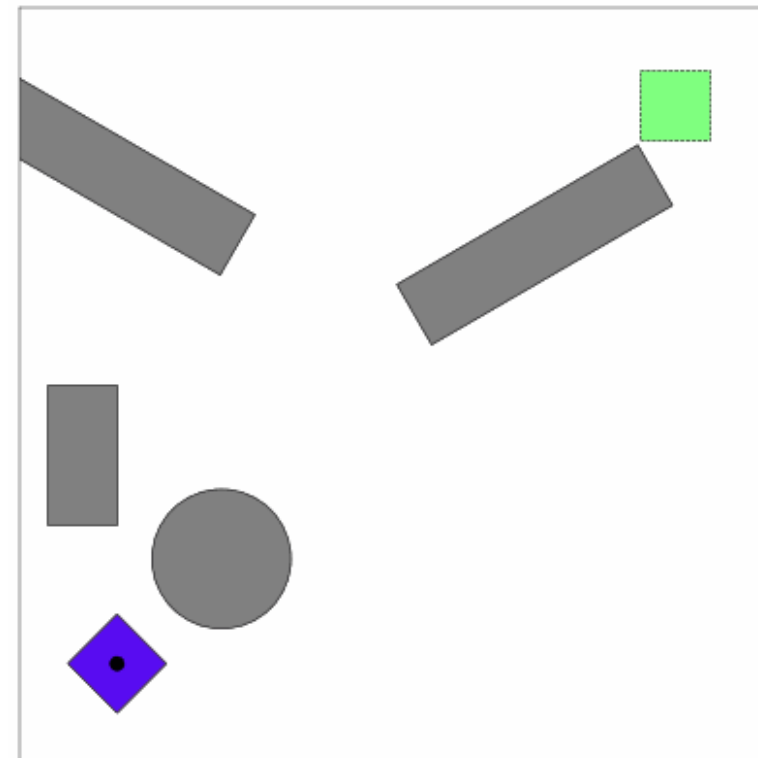
Check Distance = 5.0



Check Distance = 1.0



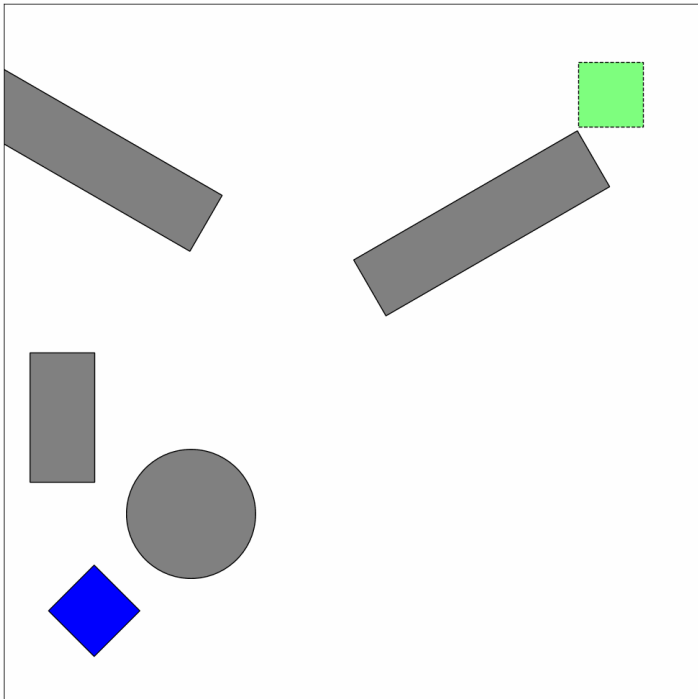
Check Distance = 0.2



Post-Processing with Shortcuts

Repeatedly sample two points on the trajectory and check if a direct line between them is feasible (rewire if so)

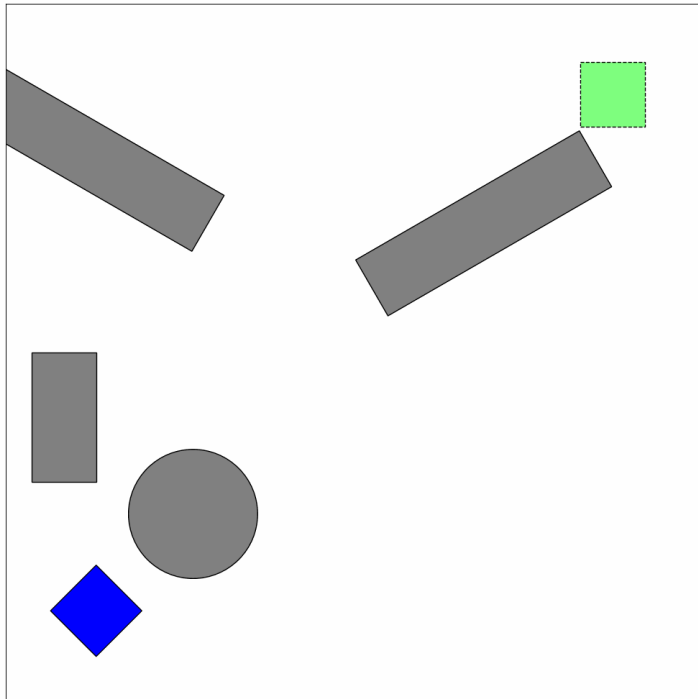
Attempts = 0



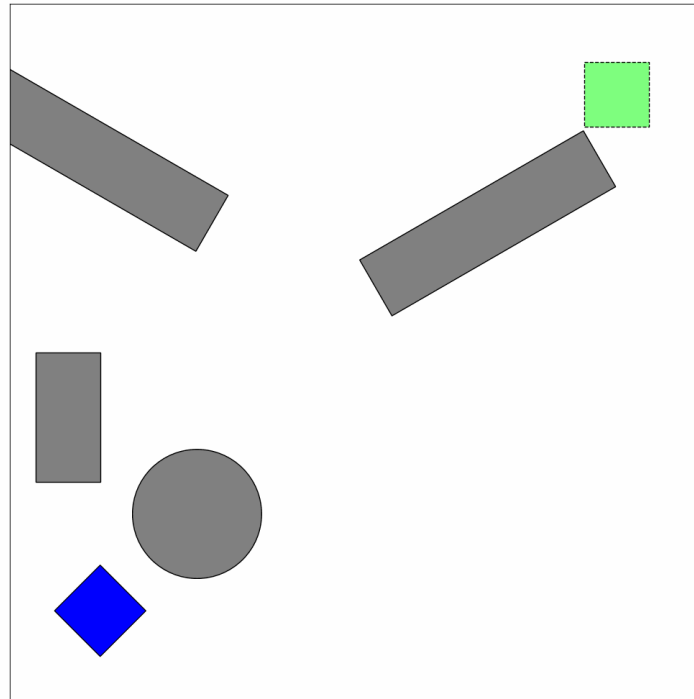
Post-Processing with Shortcuts

Repeatedly sample two points on the trajectory and check if a direct line between them is feasible (rewire if so)

Attempts = 0



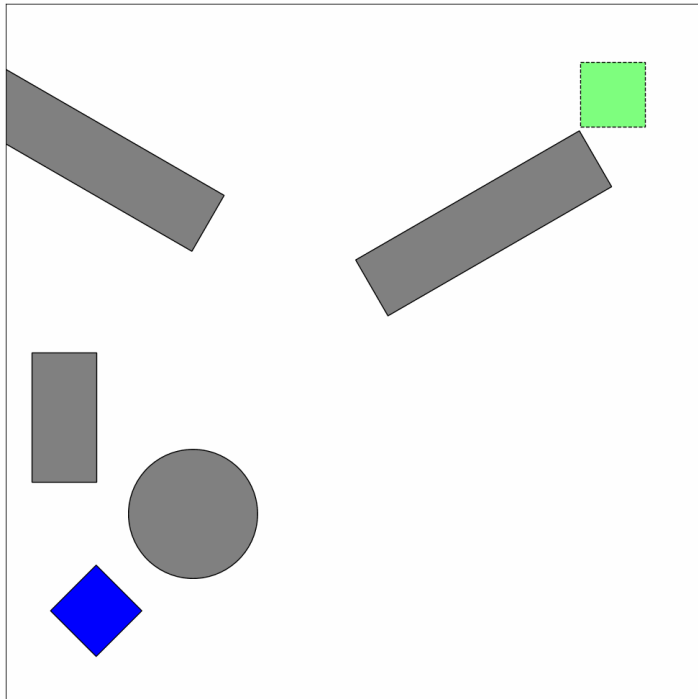
Attempts = 100



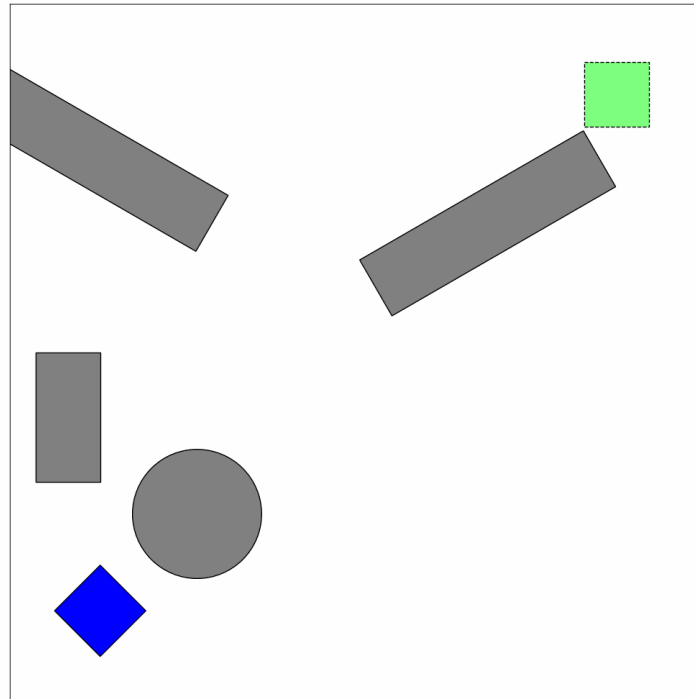
Post-Processing with Shortcuts

Repeatedly sample two points on the trajectory and check if a direct line between them is feasible (rewire if so)

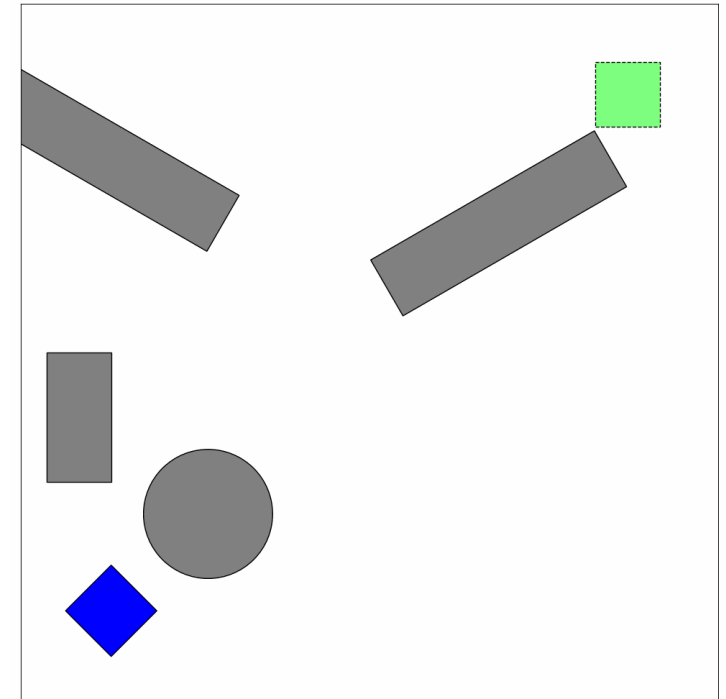
Attempts = 0



Attempts = 100



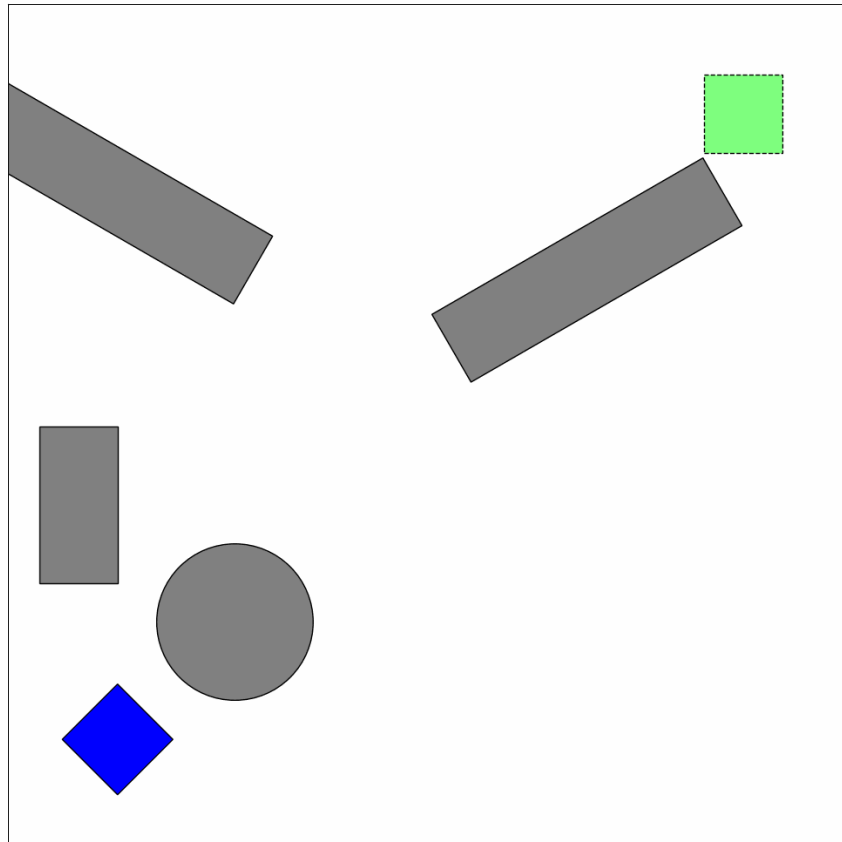
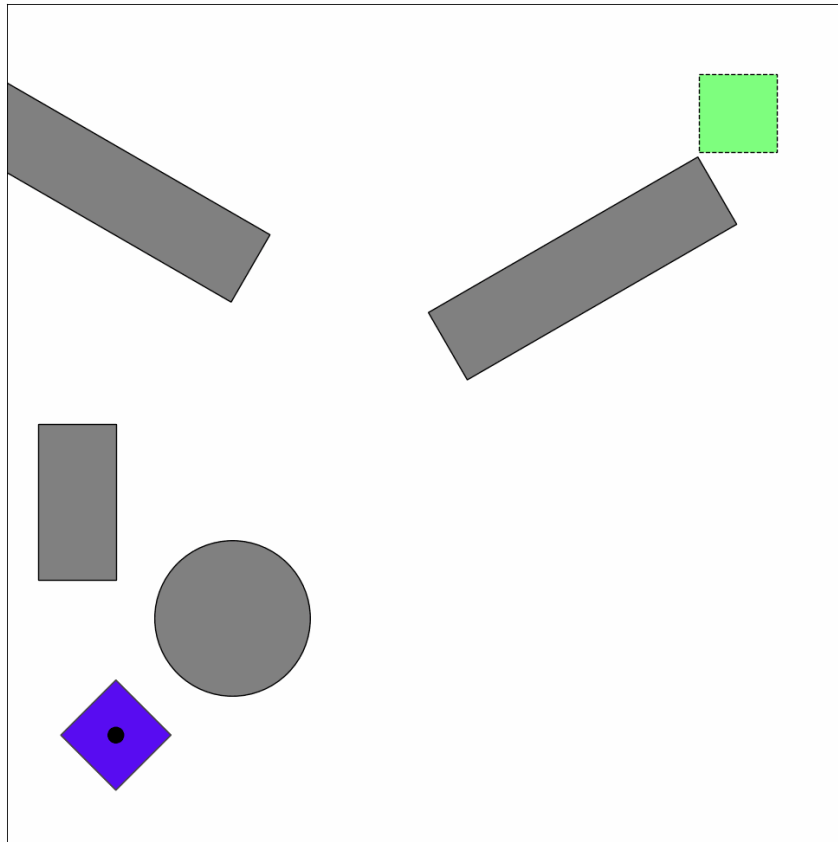
Attempts = 10000



Bidirectional RRT

Does not work for underactuated systems

Grow two trees: one from start, the other from goal



Multi-Query Motion Planning

- RRT grows tree once, from initial to goal
- After the robot moves, need to start from scratch
- Can we build a representation once, up front, instead?
- Need a *graph* instead of a *tree*

Probabilistic Roadmaps (PRMs)

BUILDPRM($\mathcal{X}, f$)

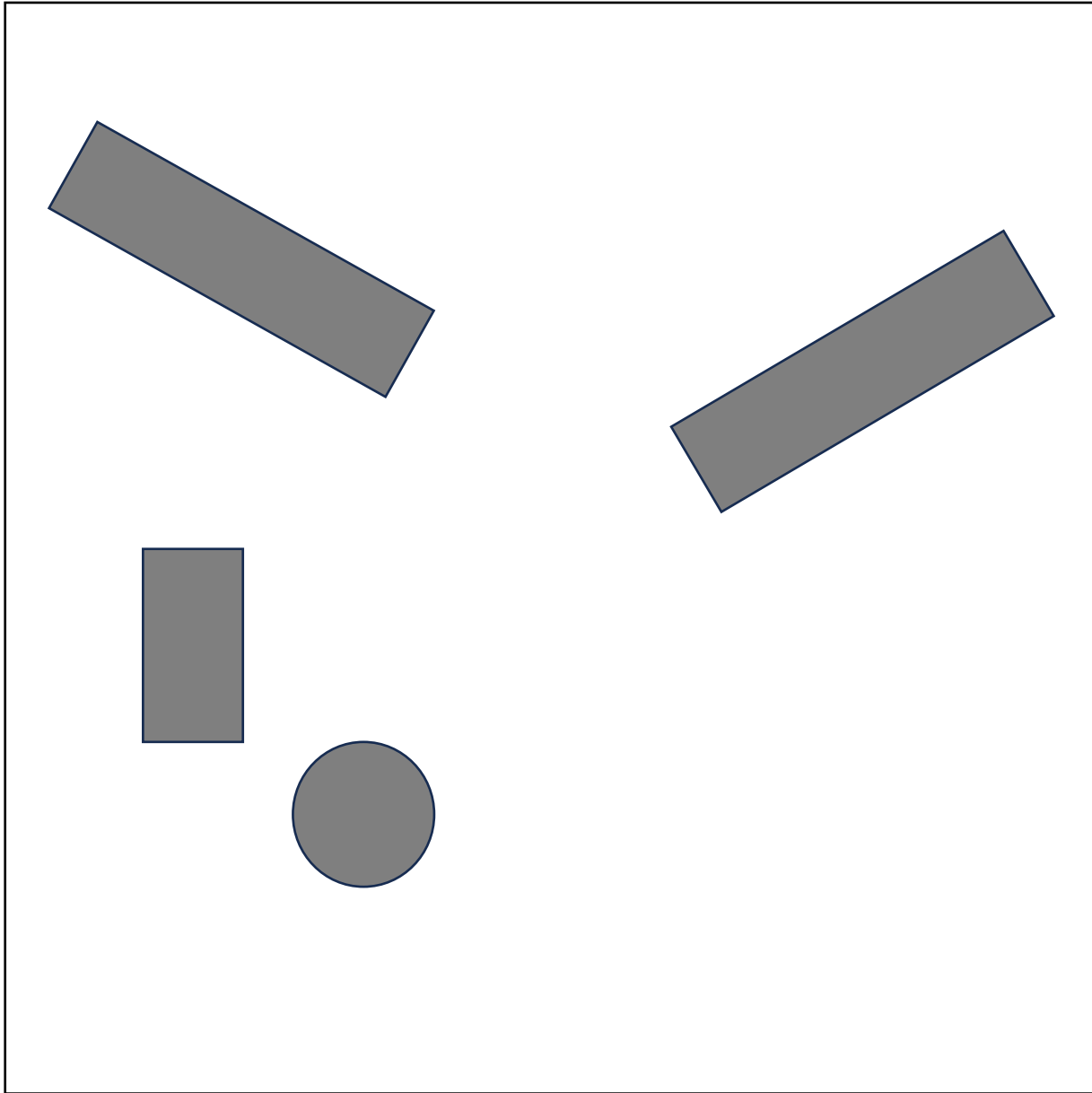
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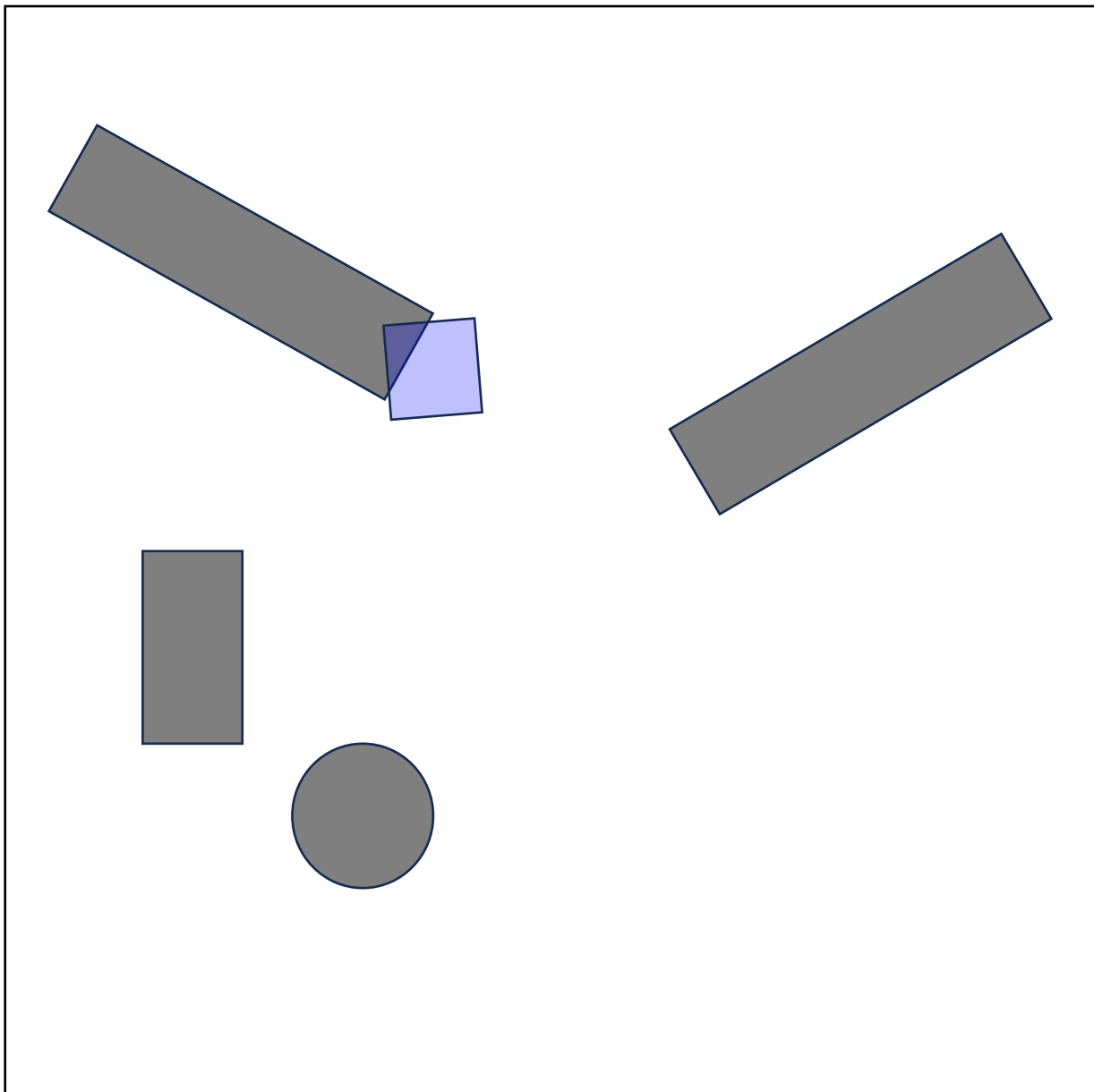
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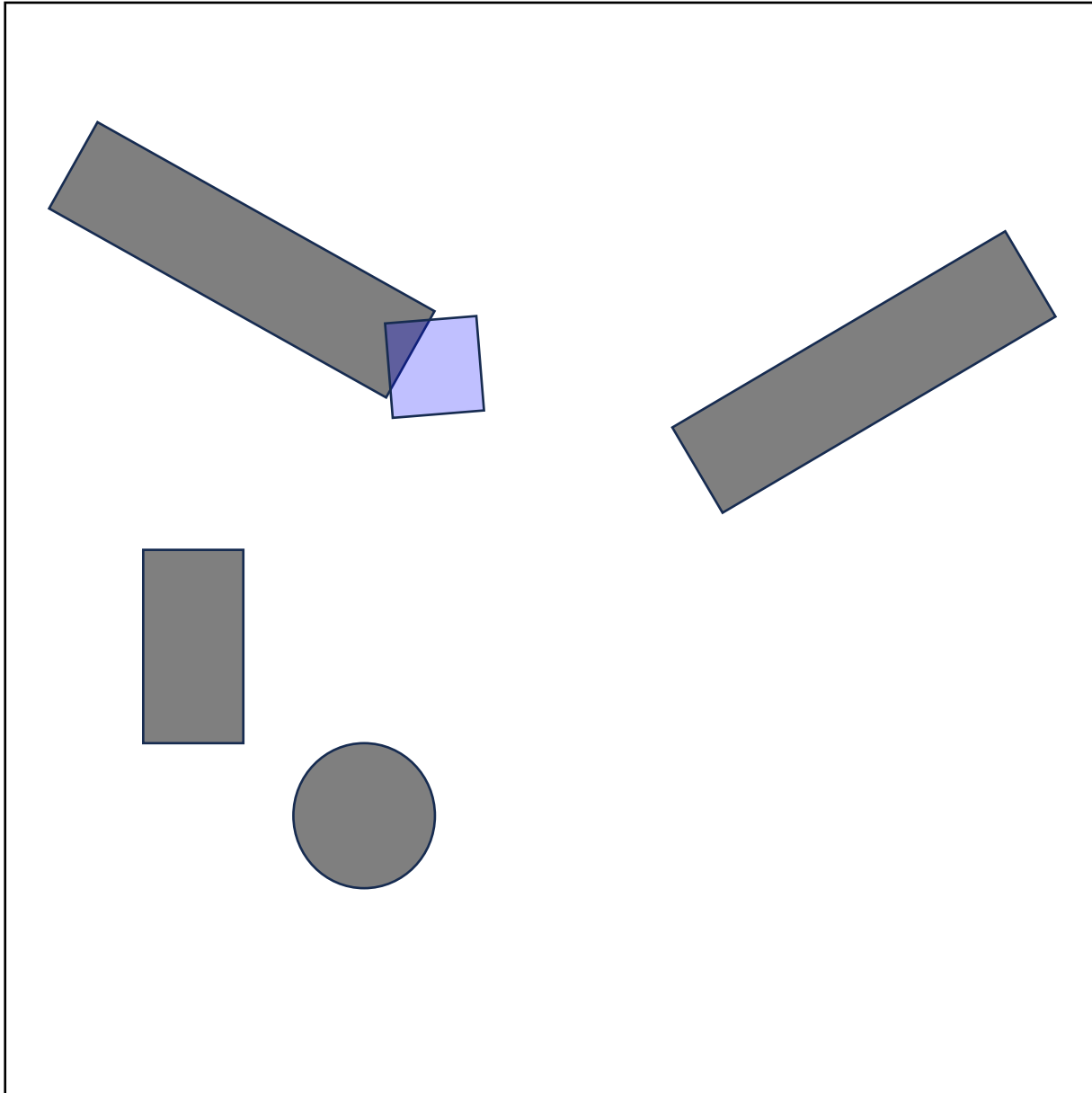
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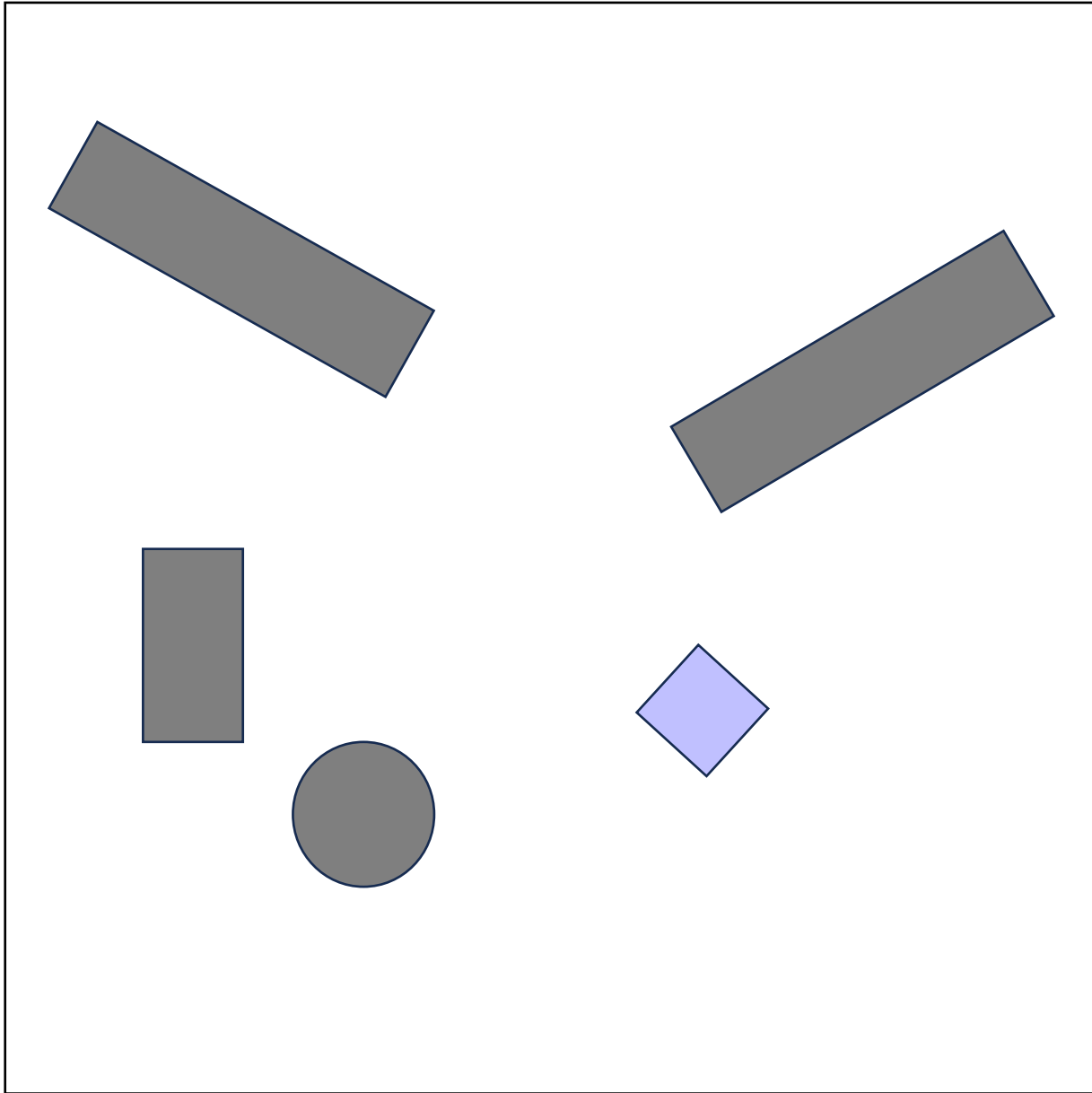
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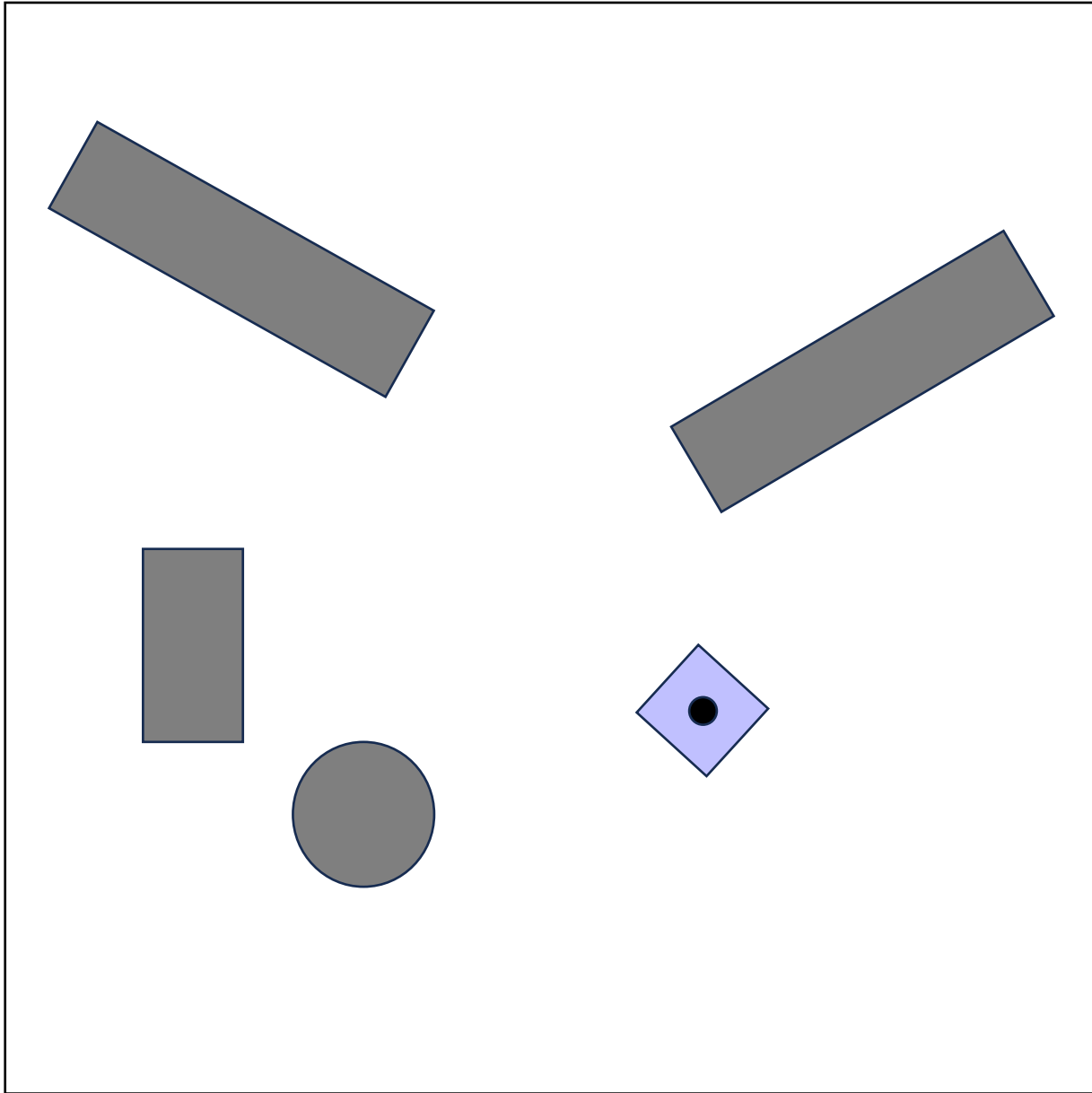
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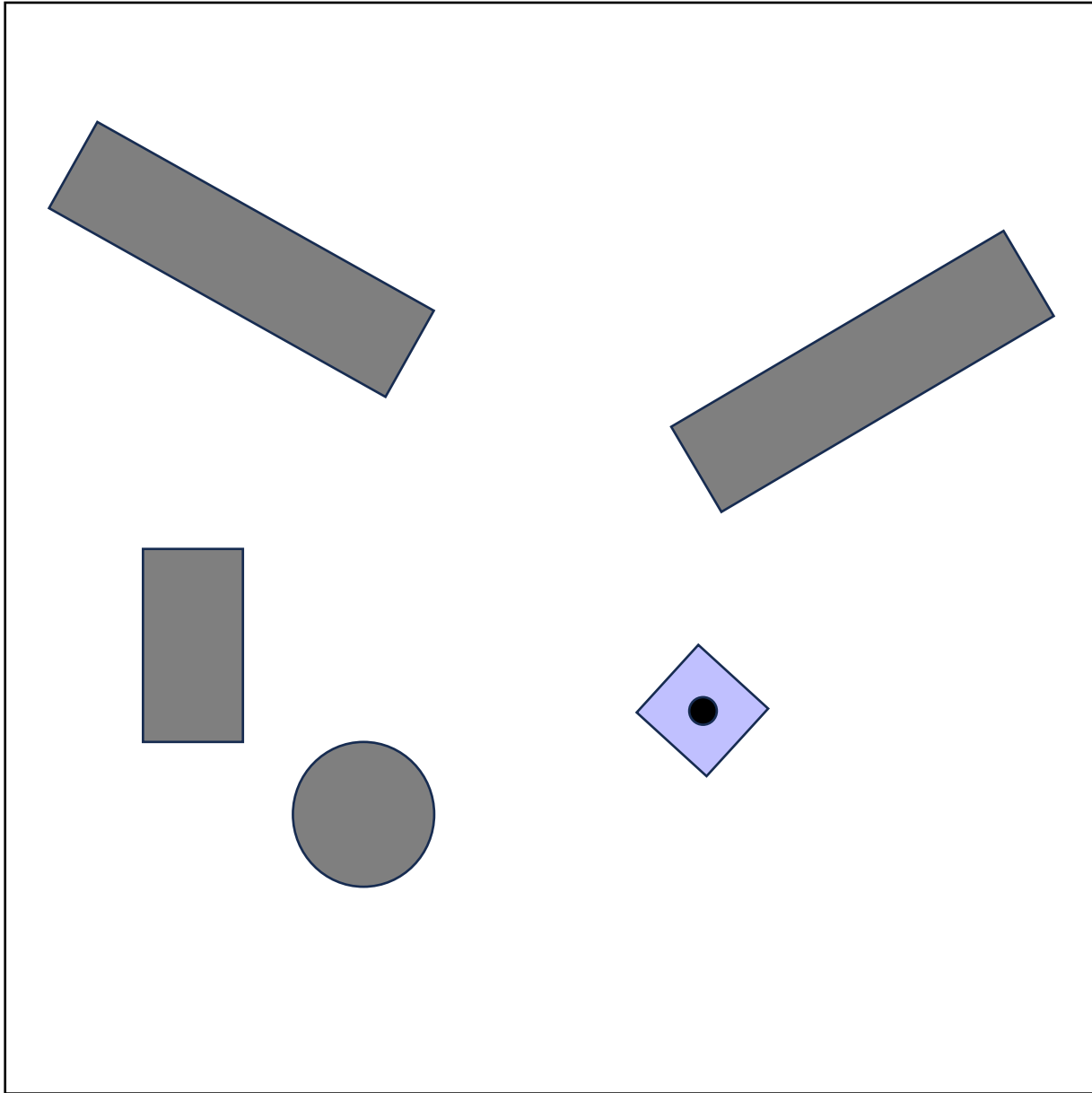
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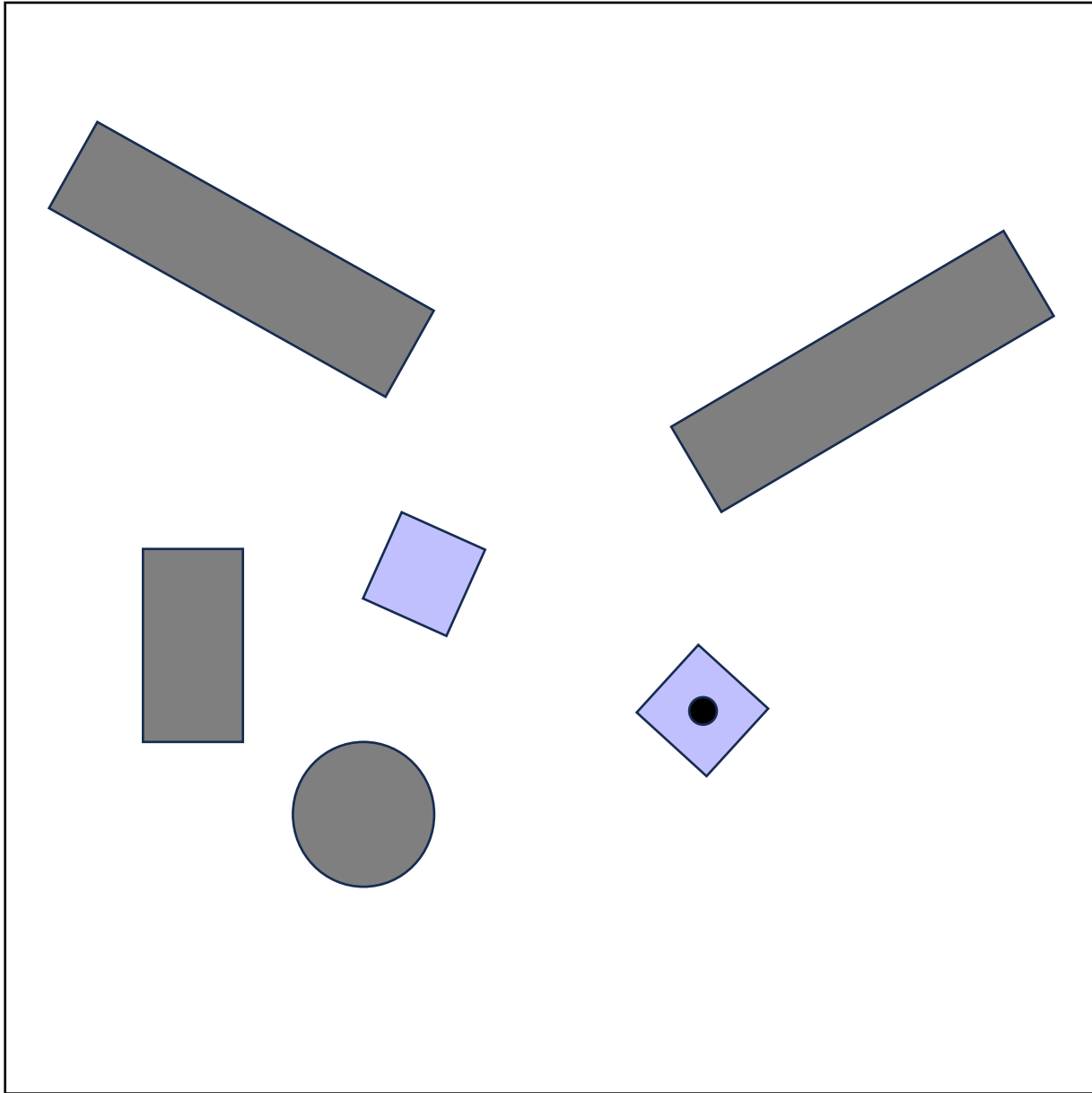
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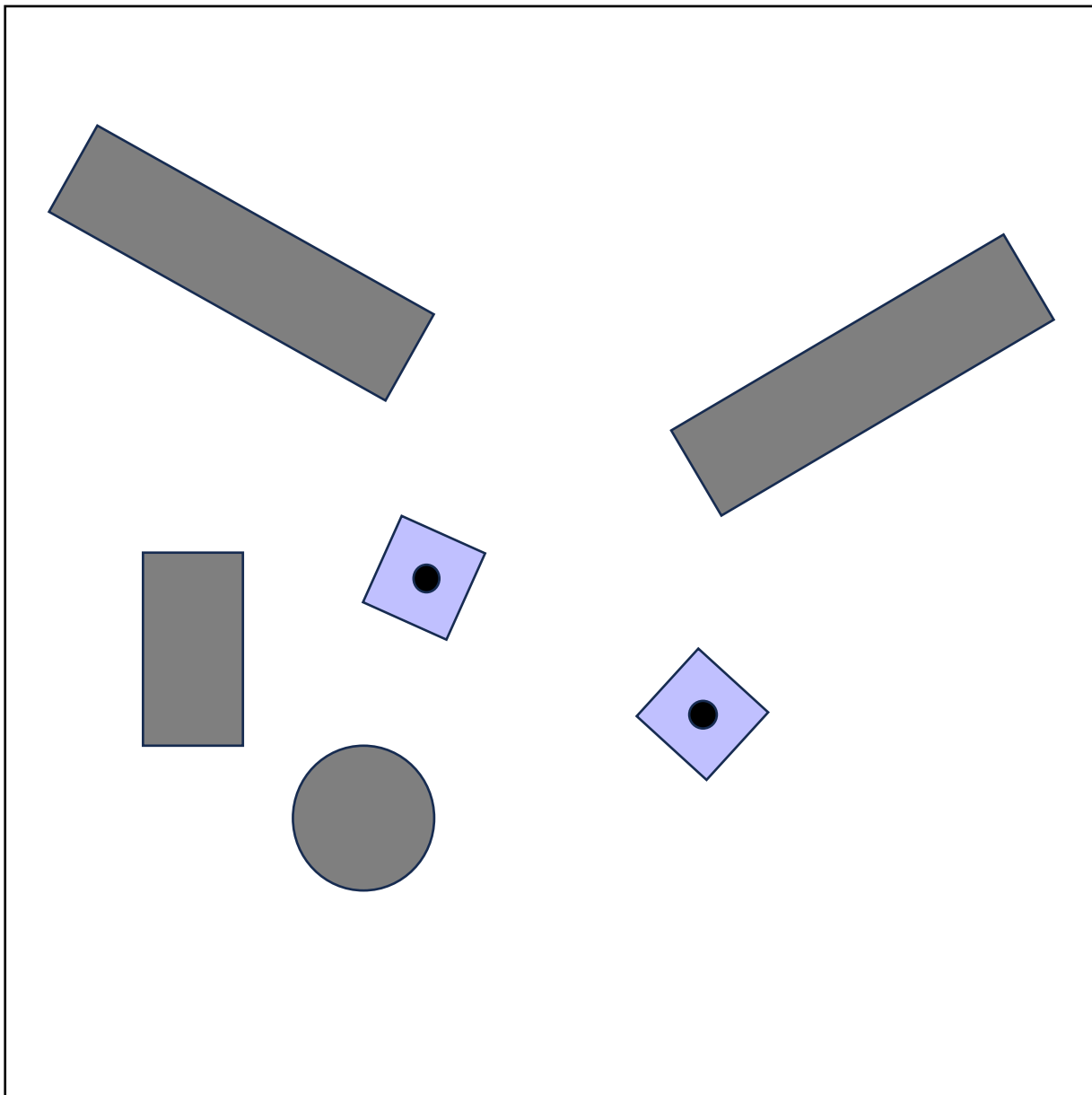
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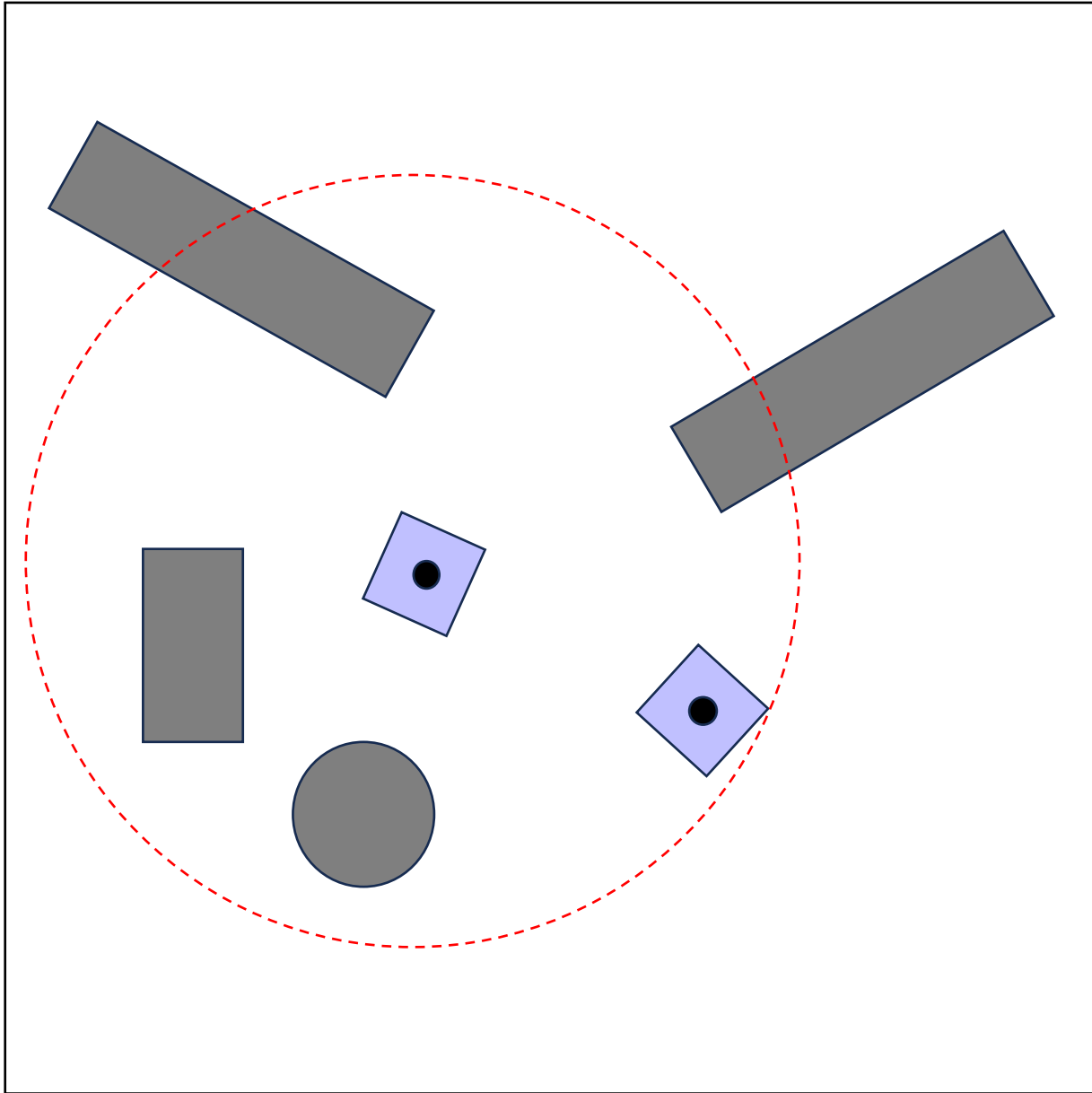
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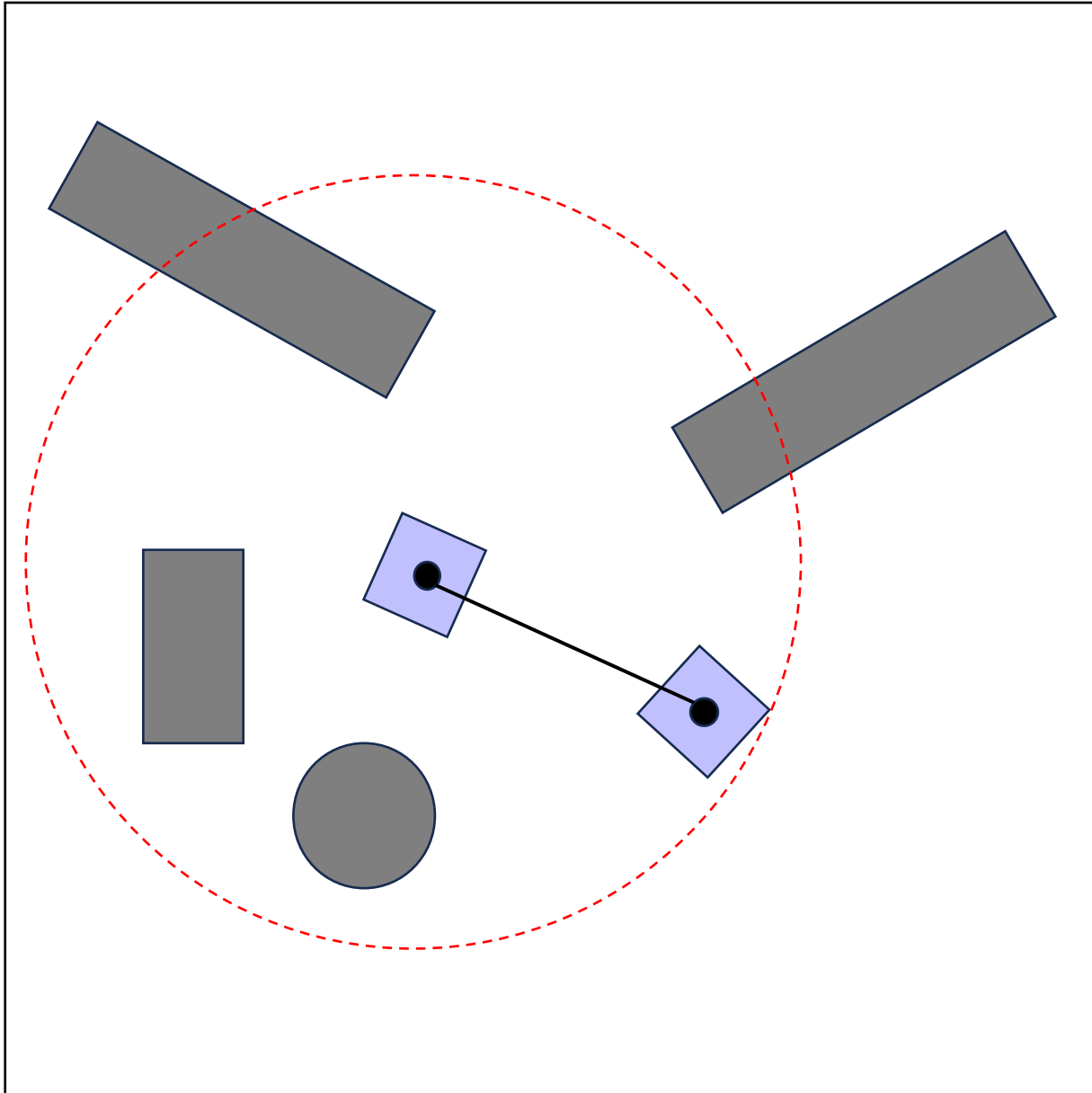
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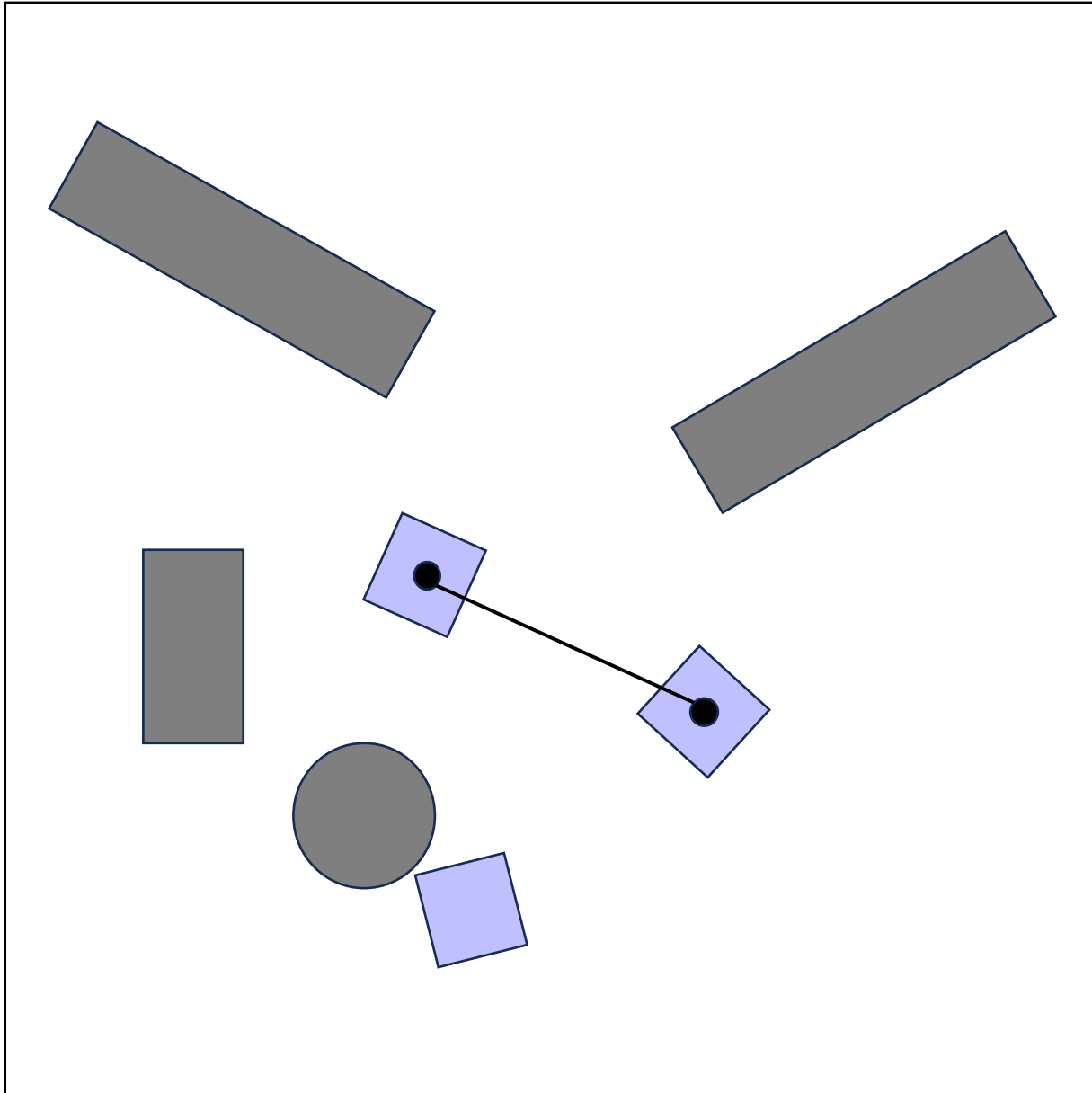
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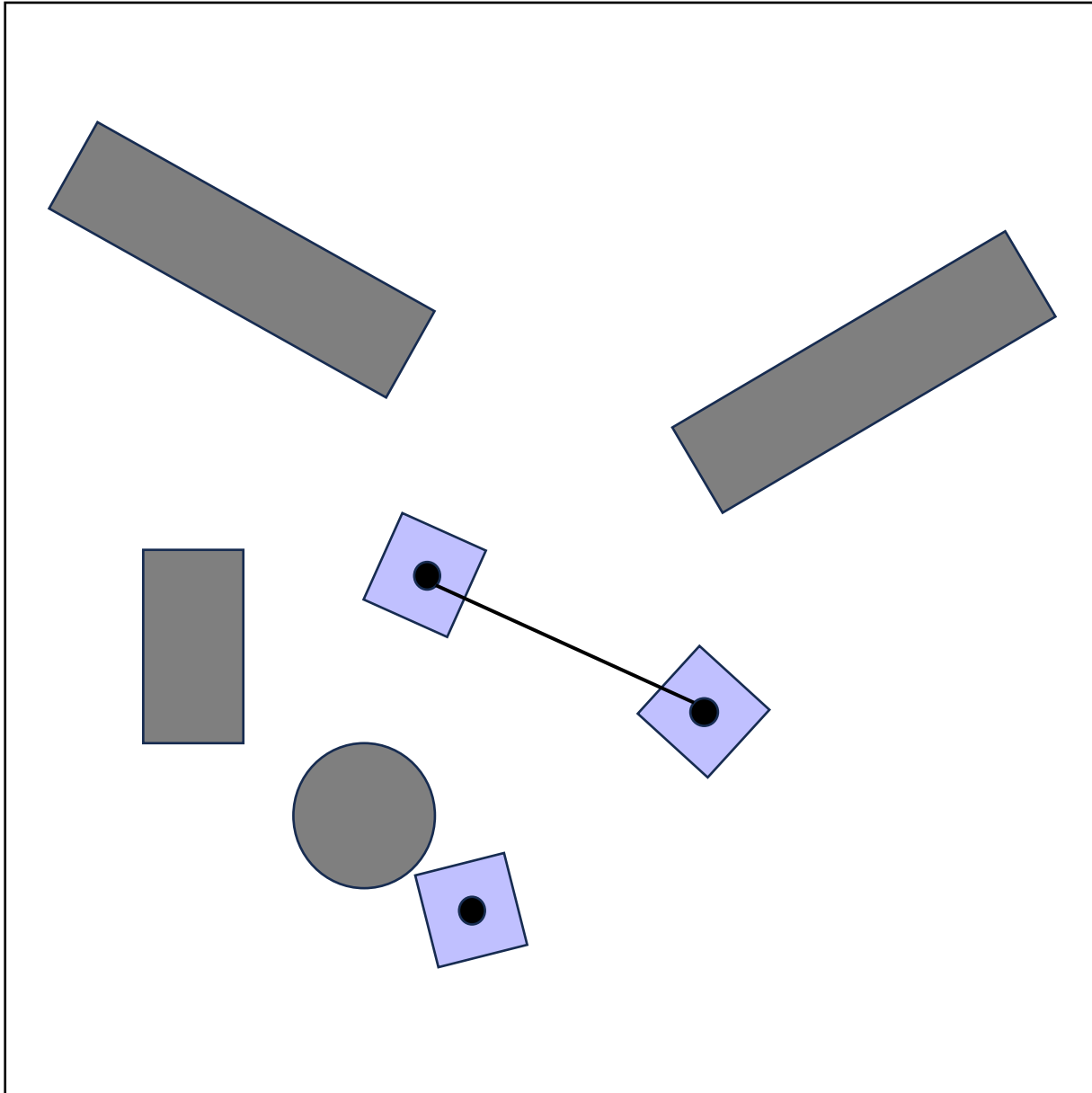
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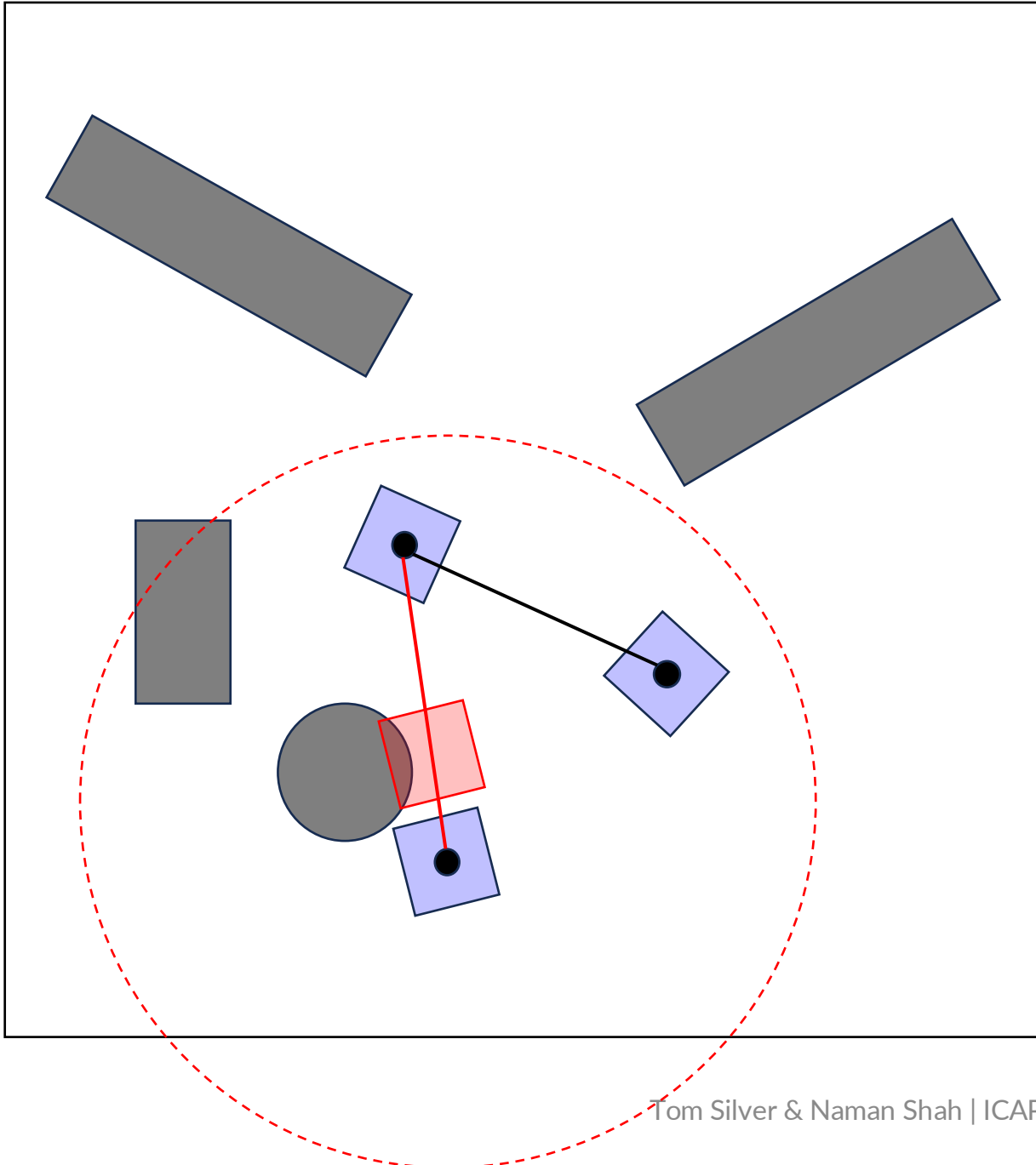
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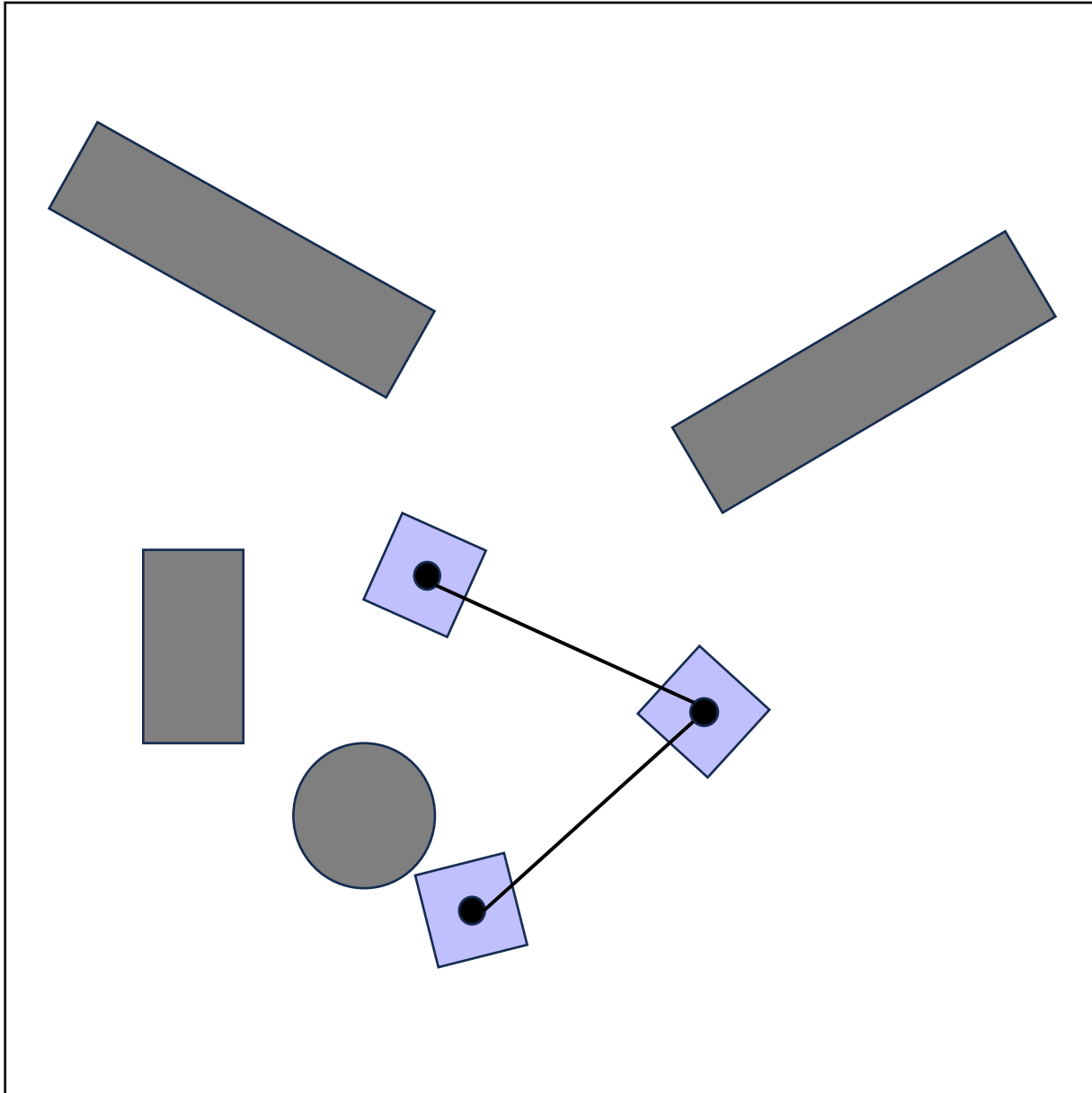
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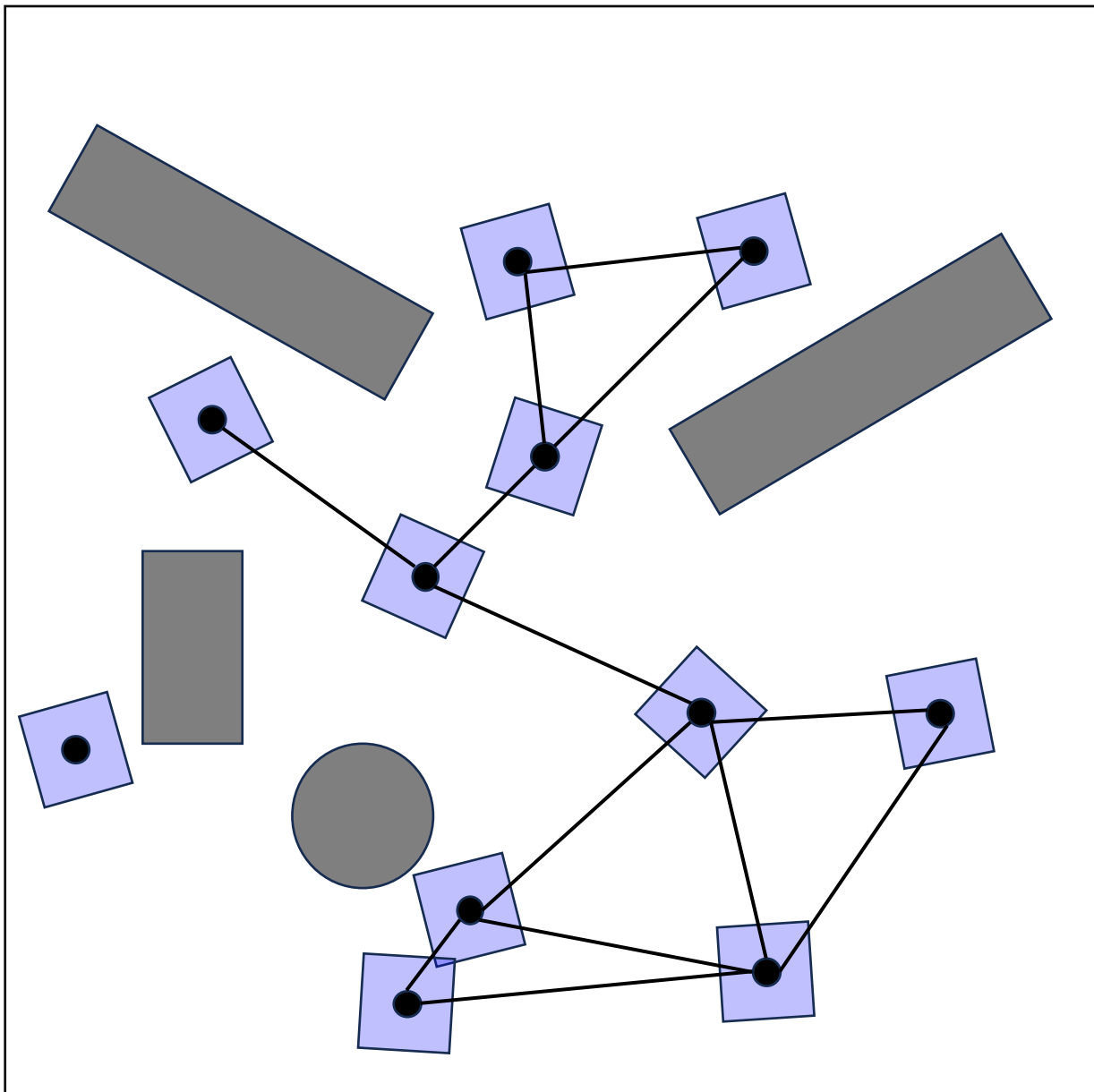
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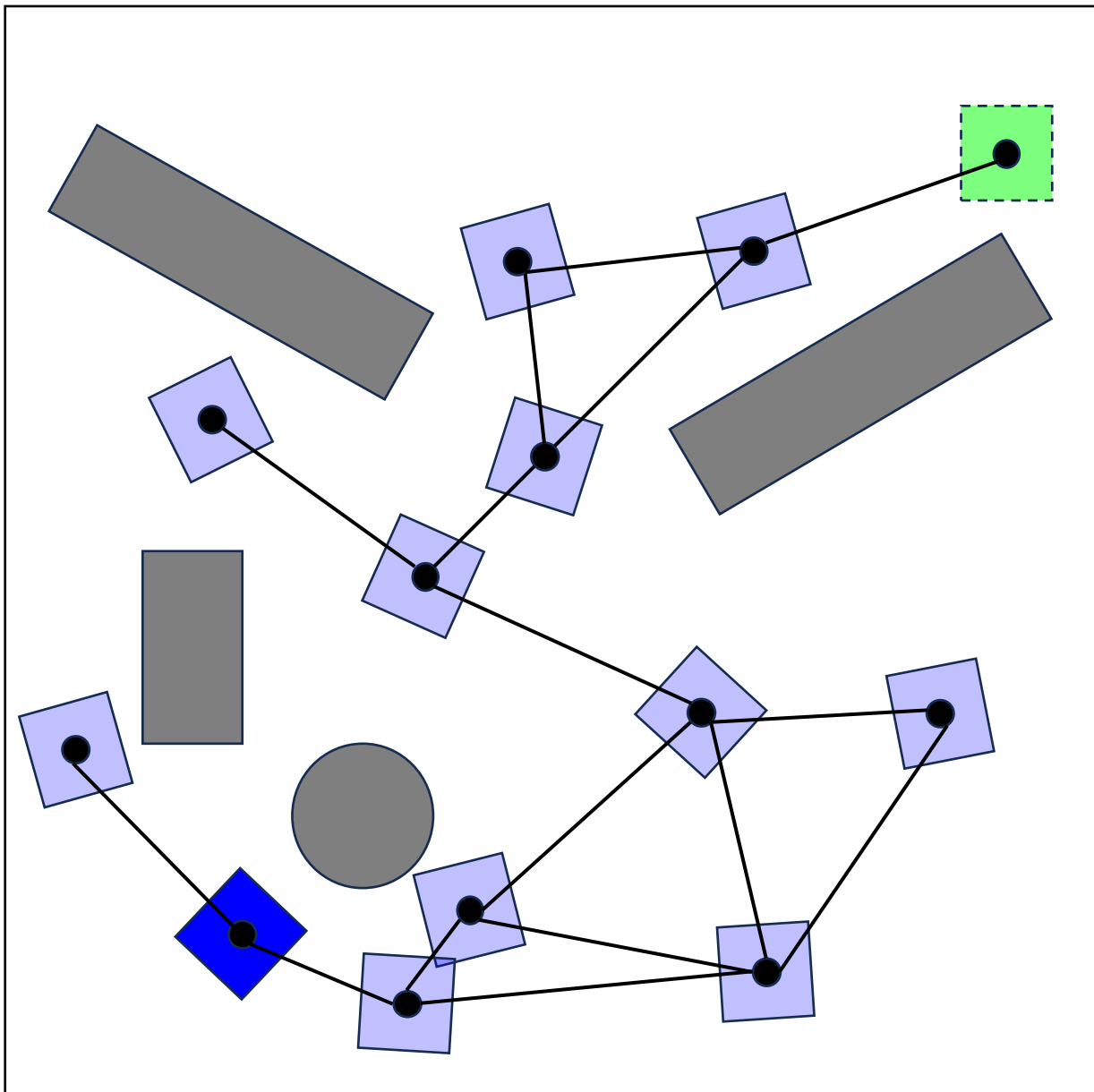
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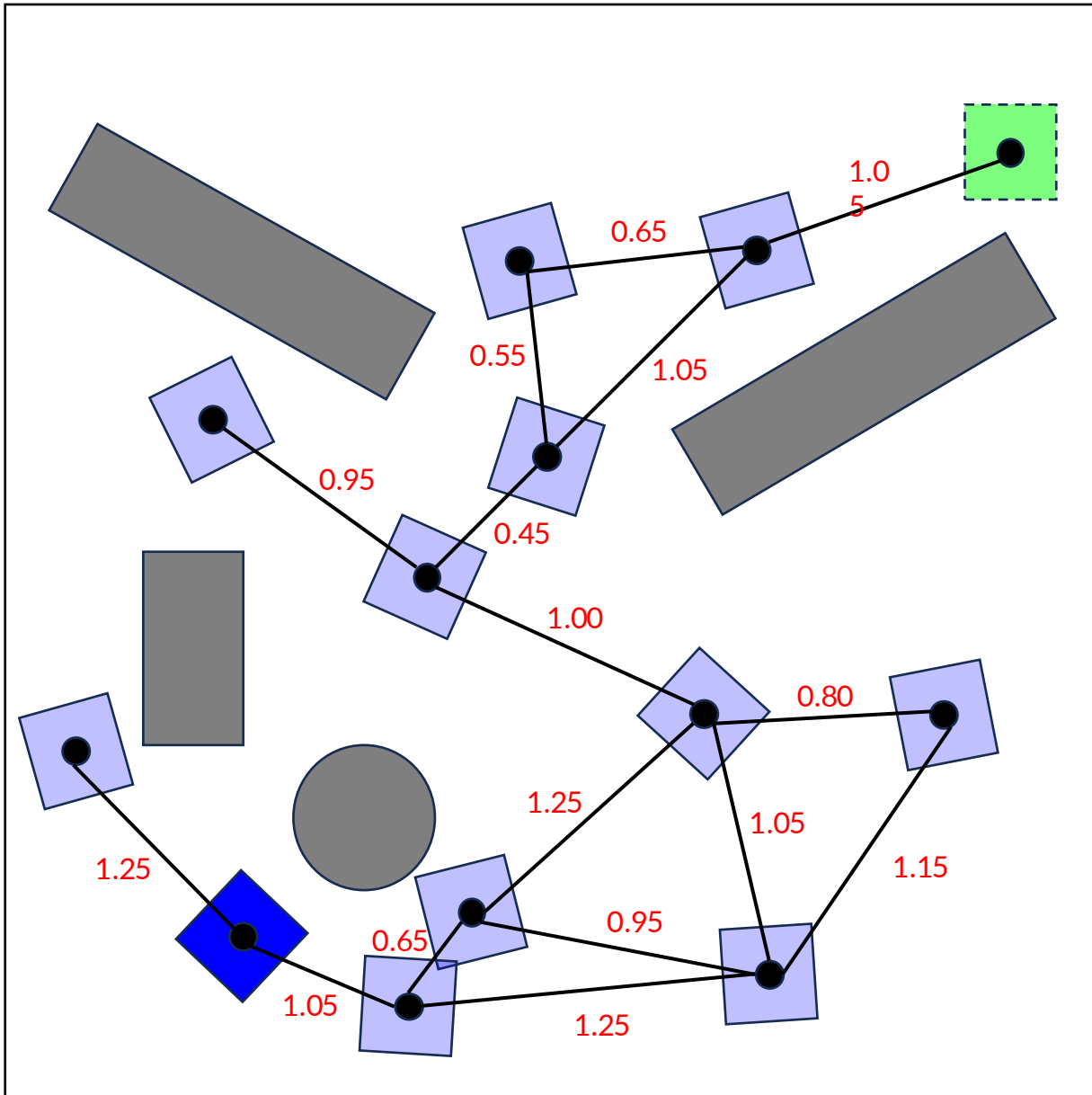
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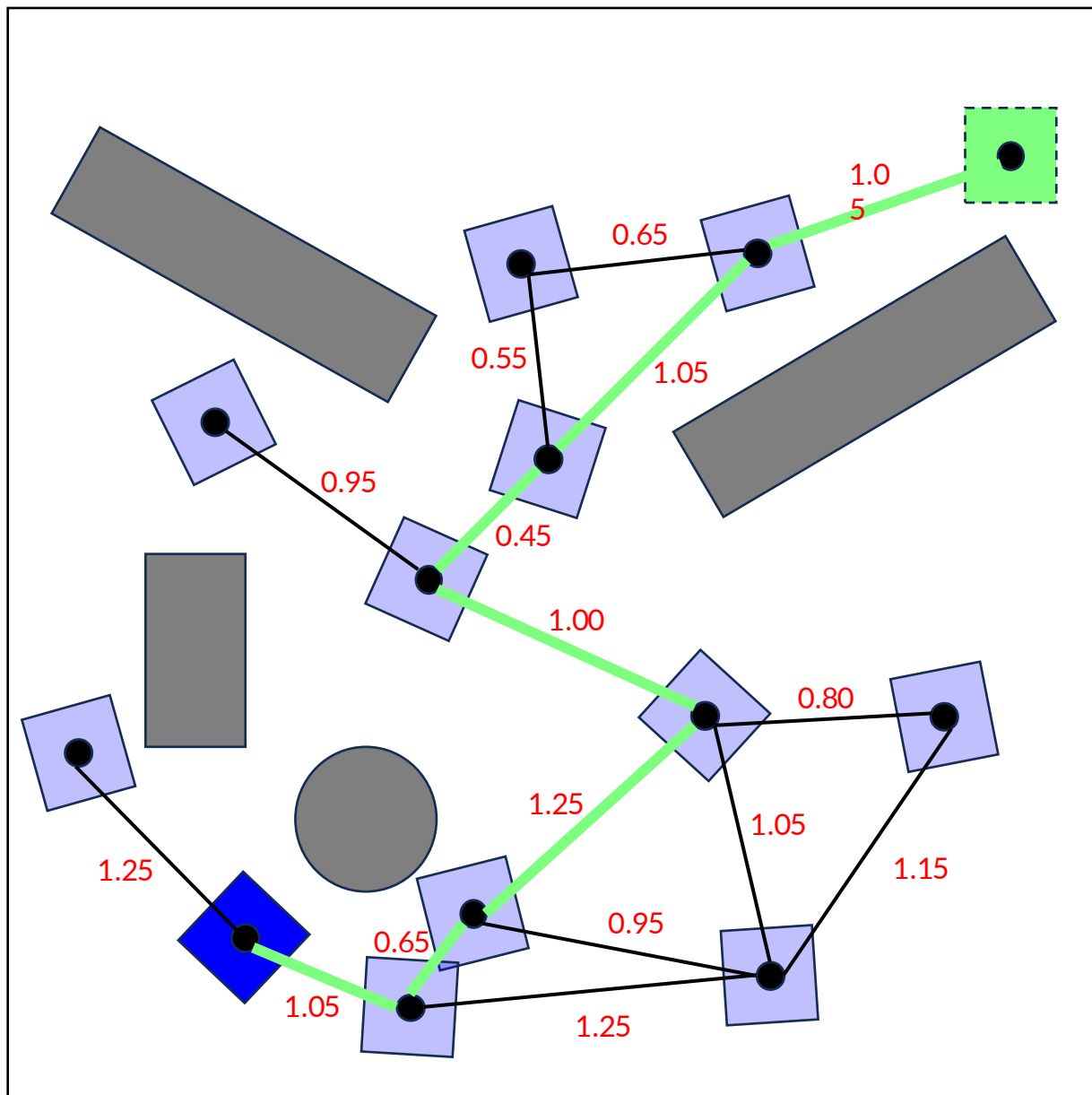
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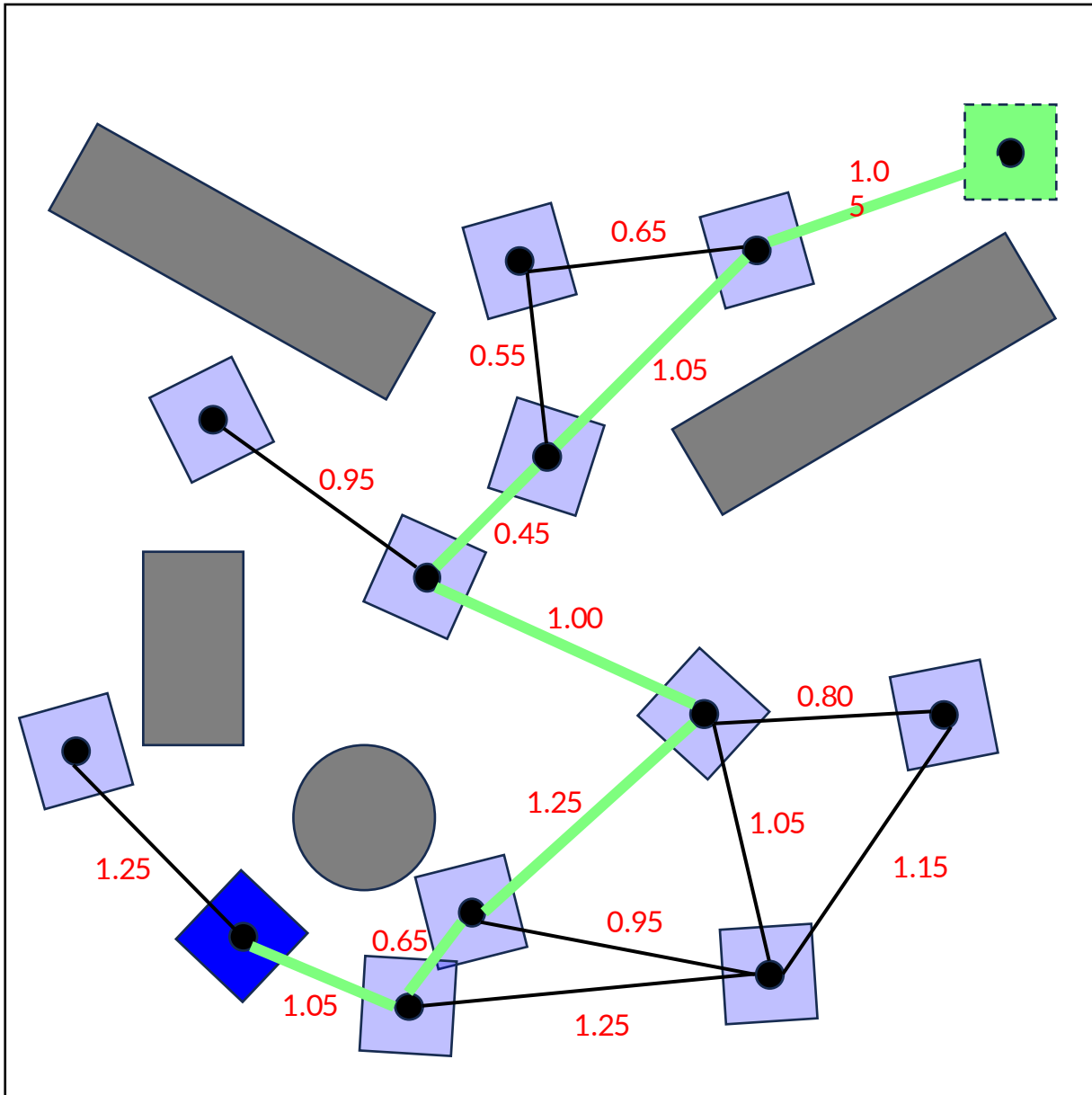
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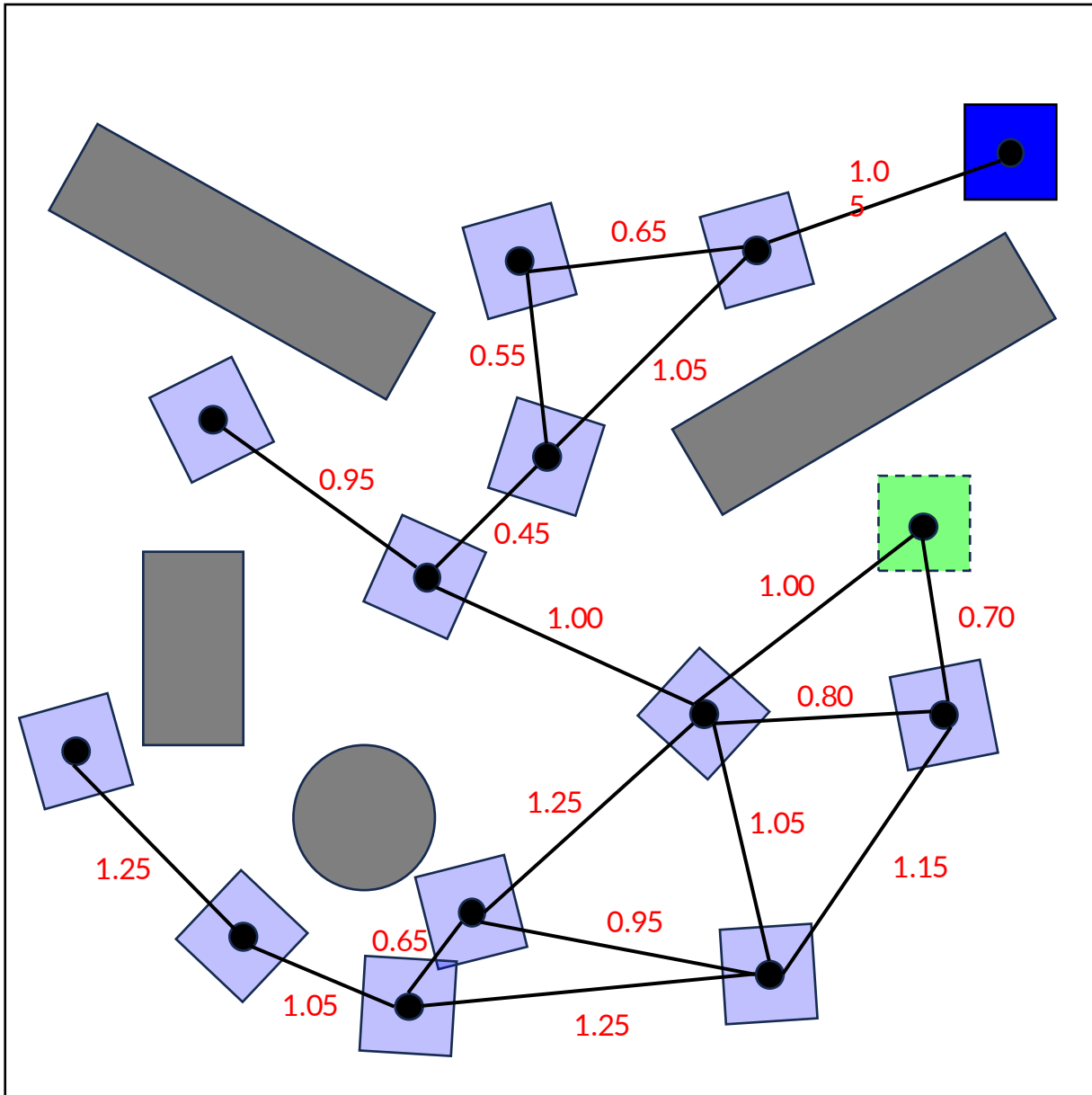
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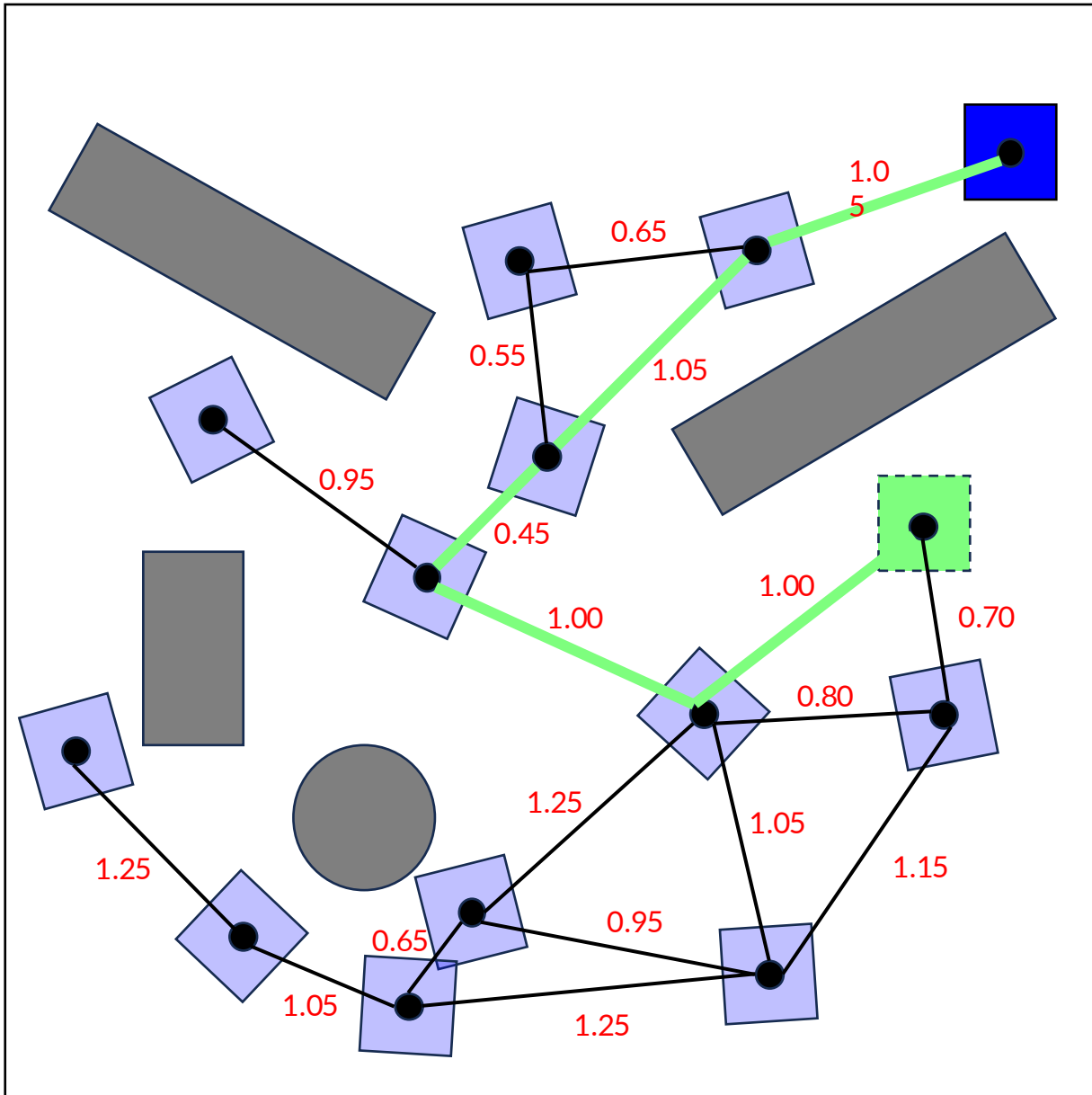
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2 goalNode = UpdatePRM( $x_g$ , graph,  $f$ )
3 nodePath = graphSearch(initNode, goalNode)
4 return finish(nodePath)

```



BUILDPRM($\mathcal{X}, f$)

```

1 graph = UndirectedGraph()
2 repeat:
3     // Sample from configuration space
4      $x = \text{sample}(\mathcal{X})$ 
5     // Skip if not feasible
6     if not  $f(x)$ : continue
7     // Update the graph
8     UpdatePRM( $x$ , graph,  $f$ )
9 return graph

```

UPDATEPRM(x , graph, f)

```

1 newNode = addNode(graph,  $x$ )
2 for node  $\in$  getNeighbors( $x$ )
3     if pathFeasible(node.conf,  $x$ ,  $f$ ):
4         addEdge(graph, node, newNode)
5 return newNode

```

Reuse graph for
new query

QUERYPRM(x_0, x_g , graph, f)

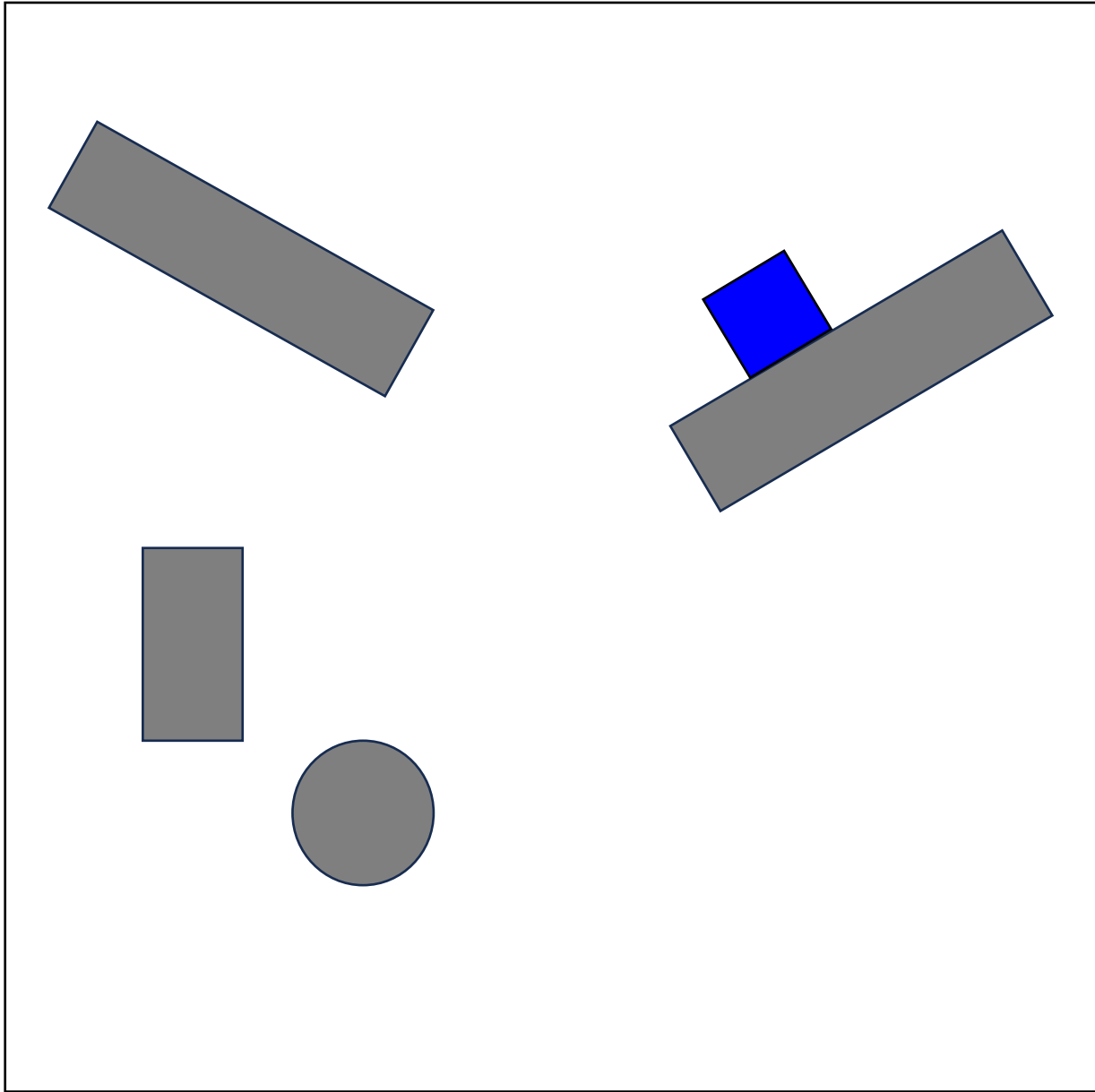
```

1 initNode = UpdatePRM( $x_0$ , graph,  $f$ )
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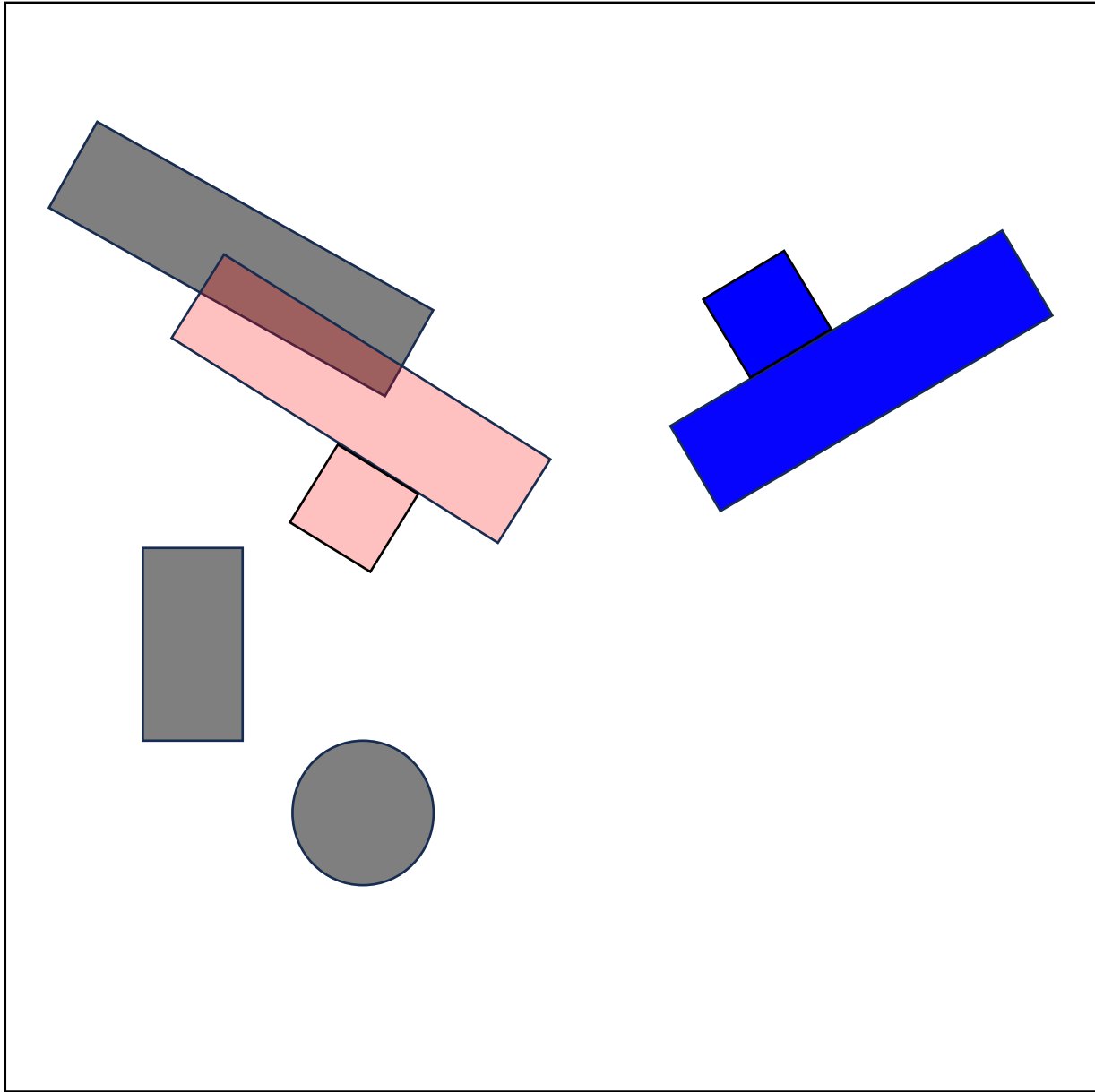
```

Morning Agenda

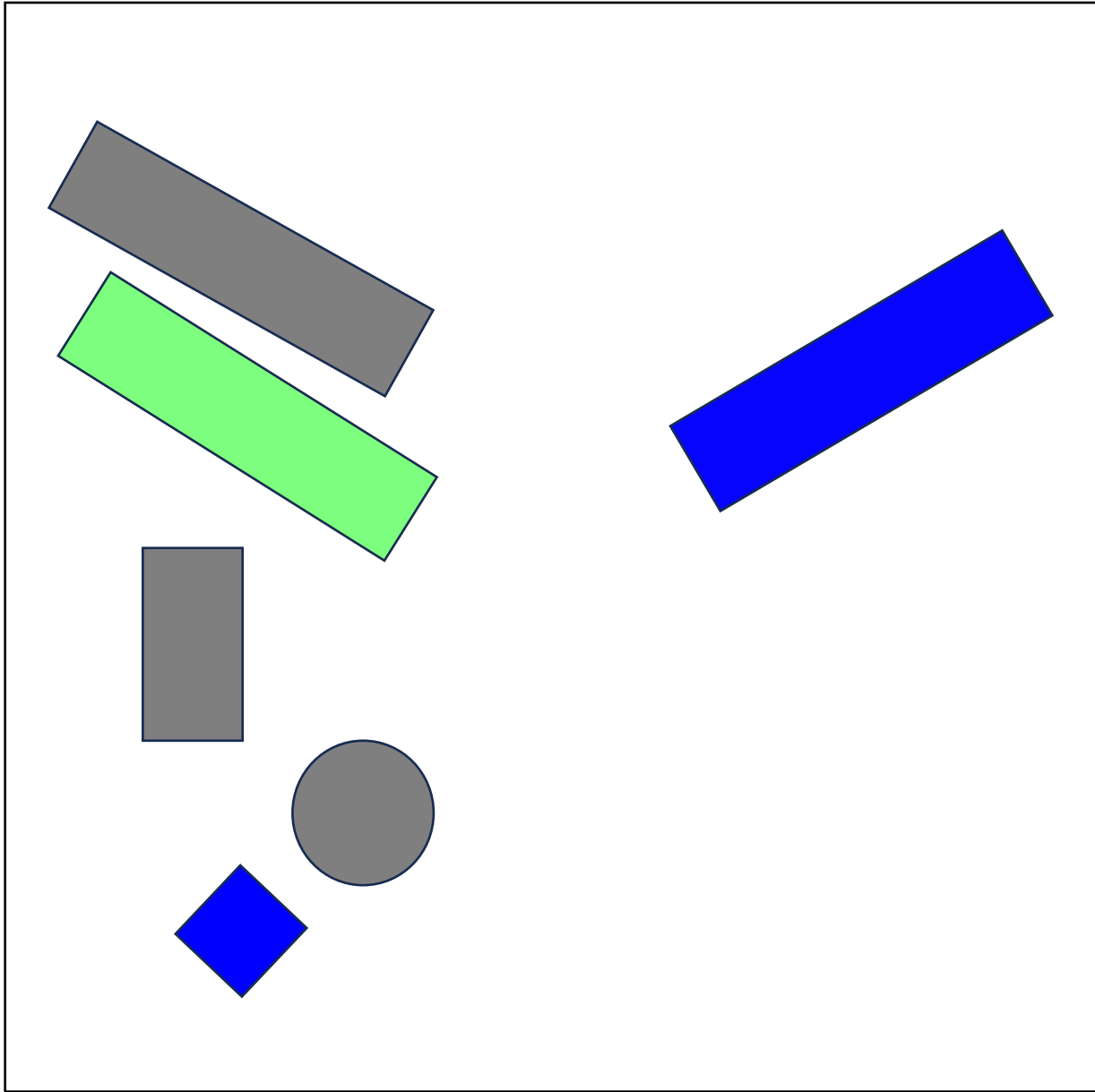
- From Task Planning to Bilevel Planning
- Introduction to Motion Planning
- **Multi-Modal Motion Planning and Beyond**
- TAMP in the Real World (TipTop)



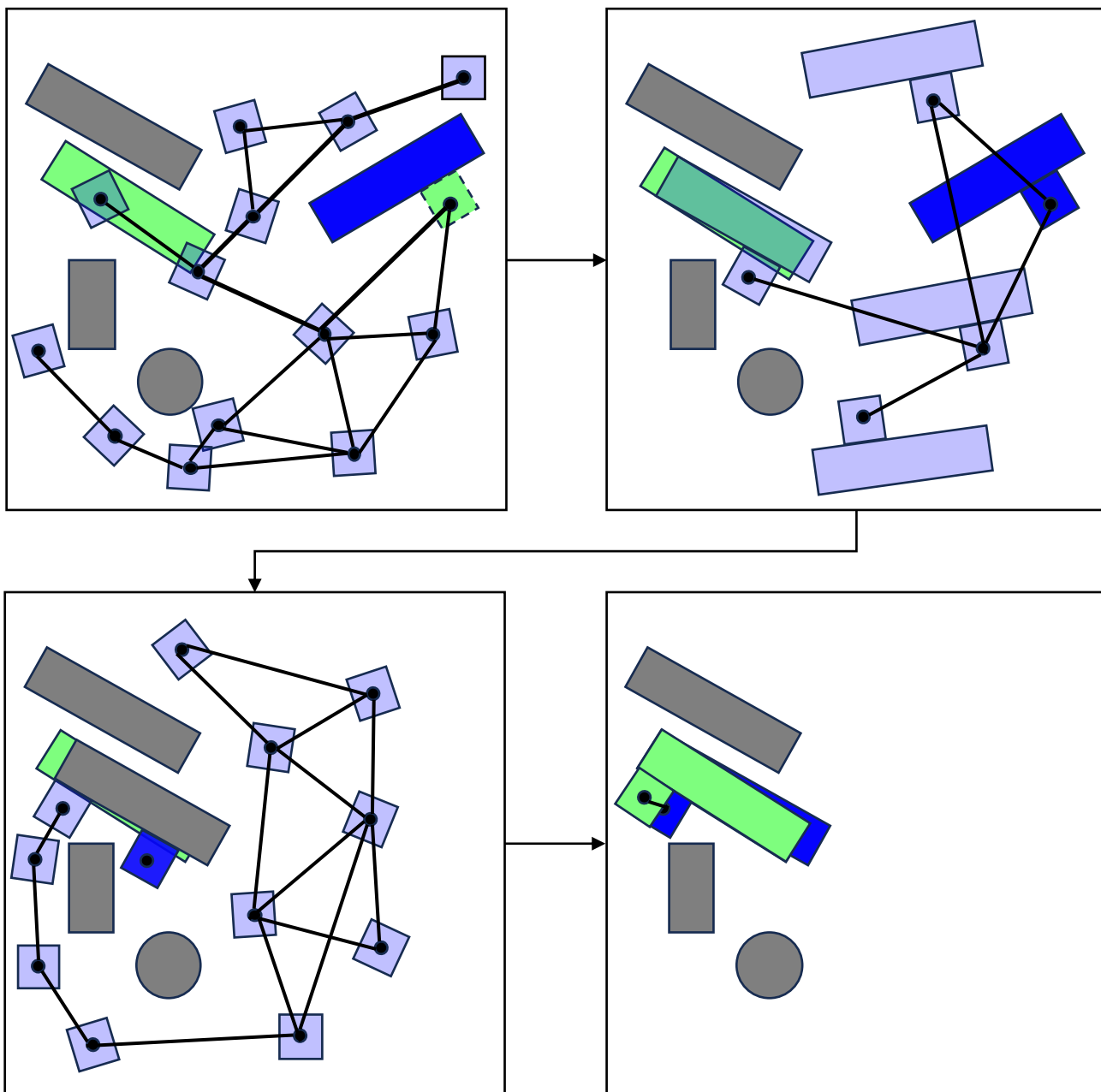
Suppose the
robot can
attach and
move
obstacles



When
attached,
feasibility
checks
change...



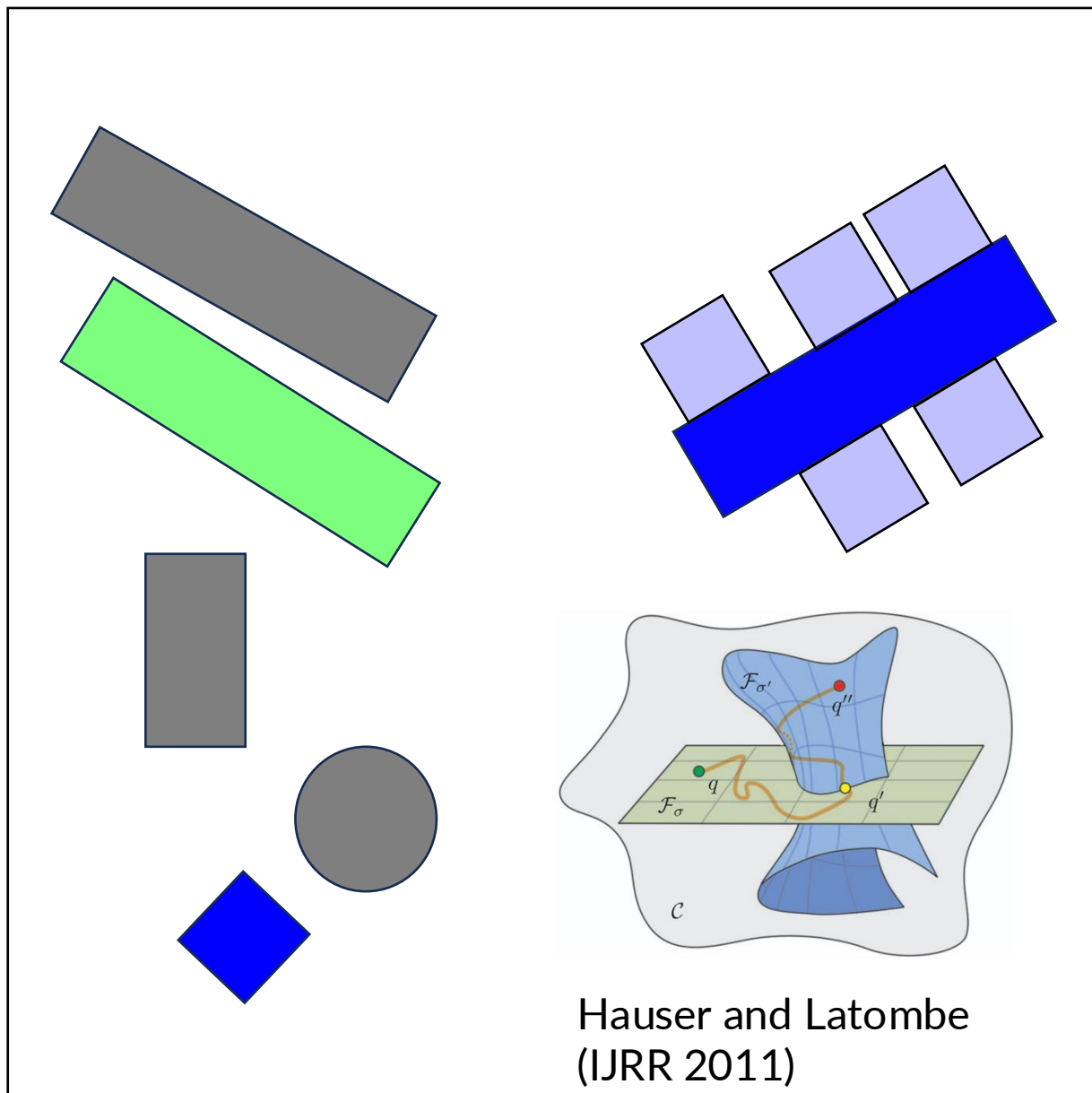
Suppose robot
can only attach
on long side.
How to reach
this goal?



Let's call each panel a
"mode" = one MP instance

Could we have used
original PRM in third panel?

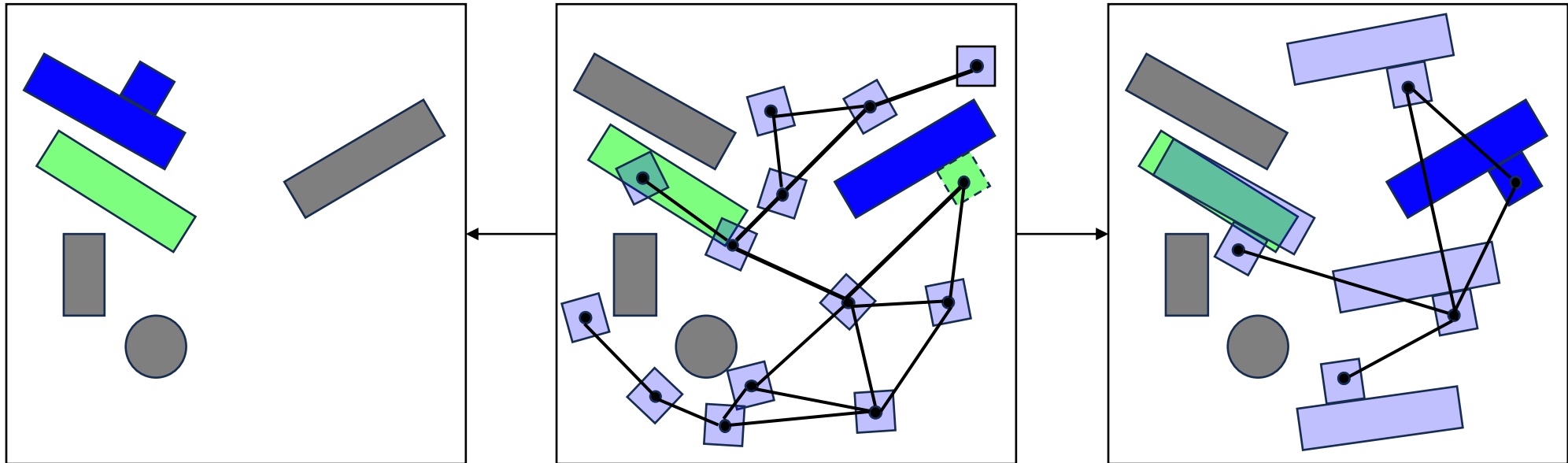
We swept something under
the rug... what's the issue?



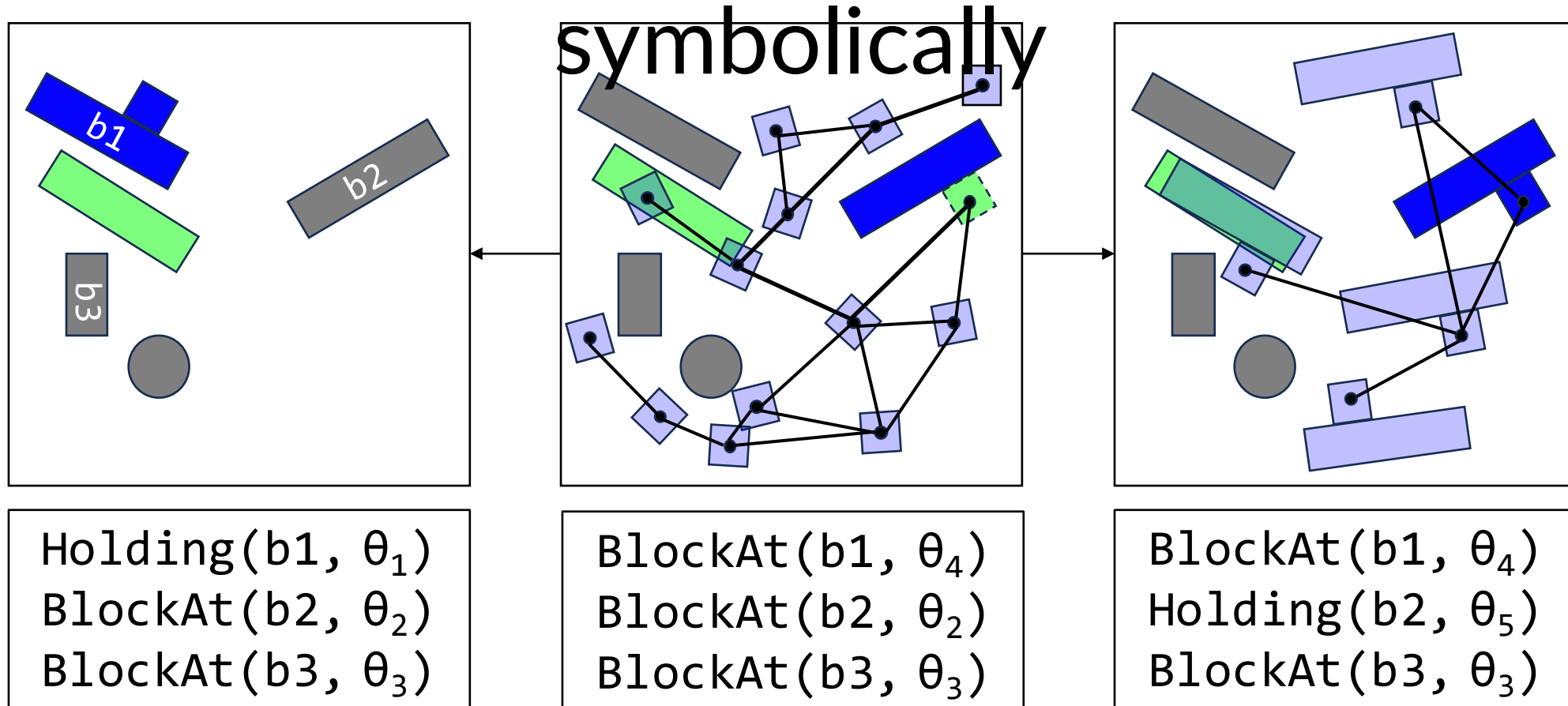
We need to be able to sample not only *within modes* (normal motion planning) but also at *mode intersections*

Reduced dimensionality!

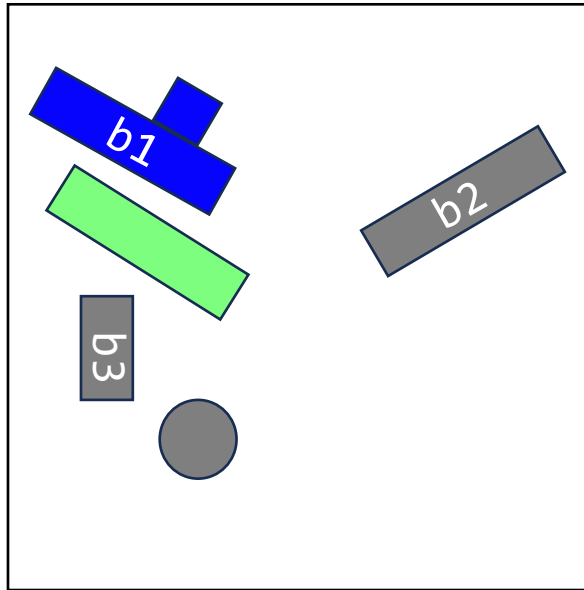
We also don't know the mode order...



Let's describe these modes

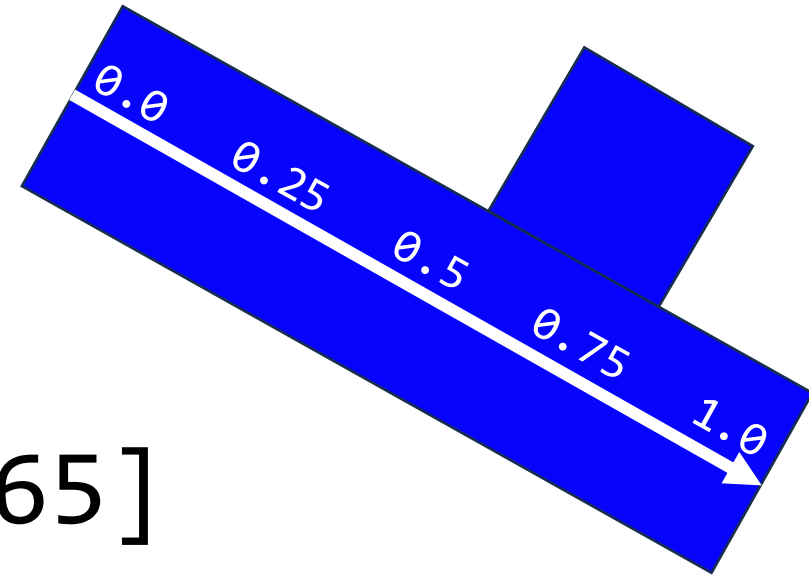


Our symbols have real numbers now



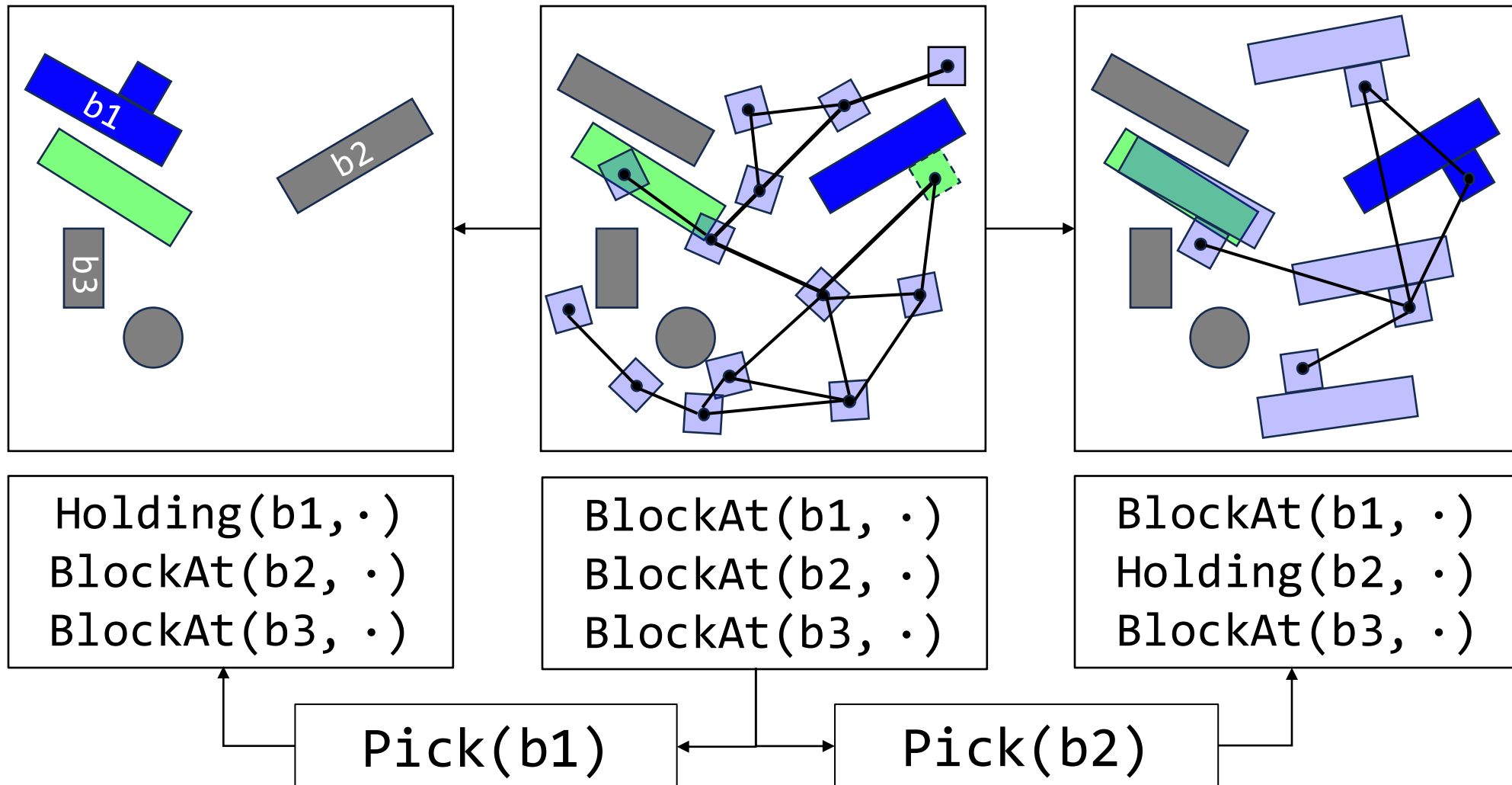
Holding(b1, θ_1)
BlockAt(b2, θ_2)
BlockAt(b3, θ_3)

$$\theta_1 = [0.65]$$



Without this, we would not
have a proper mode (MP
problem)

But, we can leave “holes” for the real numbers and task plan over modes!

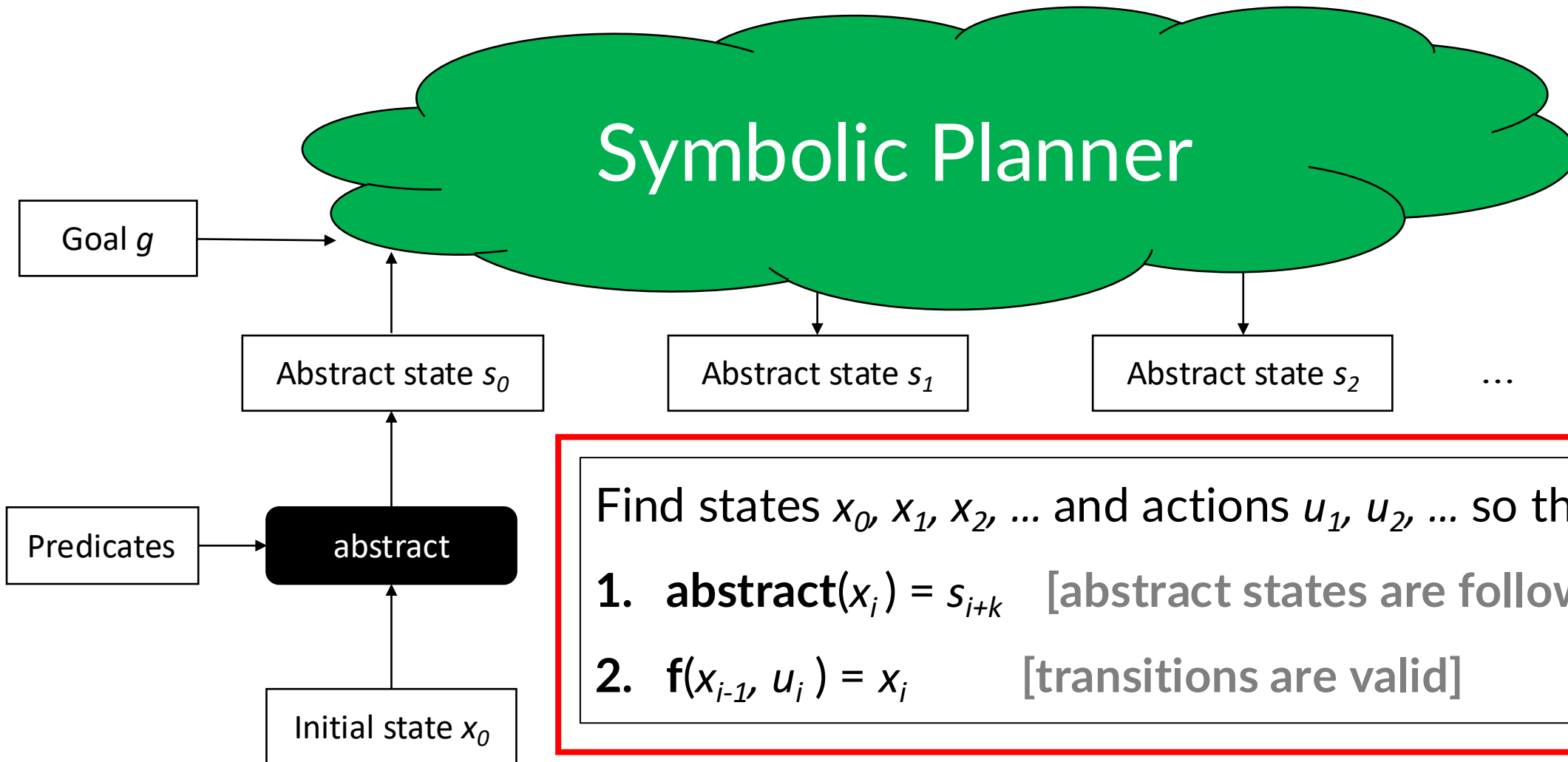


Logic-Geometric Programming

Differentiable Physics and Stable Modes for Tool-Use and Manipulation Planning

Marc Toussaint, Kelsey R Allen, Kevin A Smith & Josh Tenenbaum

In Proceedings of *Robotics: Science and Systems (R:SS 2018)*



Solve this directly (without policies)!

Morning Agenda

- From Task Planning to Bilevel Planning
- Introduction to Motion Planning
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- **TAMP in the Real World (TipTop)**

TiPToP: A Modular Open-Vocabulary Planning System for Robotic Manipulation

William Shen^{1*}, Nishanth Kumar^{1*}, Sahit Chintalapudi¹, Jie Wang², Christopher Watson², Edward S. Hu², Jing Cao¹, Dinesh Jayaraman², Leslie Pack Kaelbling¹, Tomás Lozano-Pérez¹

¹ [LIS Group, MIT CSAIL](#) ² [University of Pennsylvania](#)

*Indicates equal contribution.

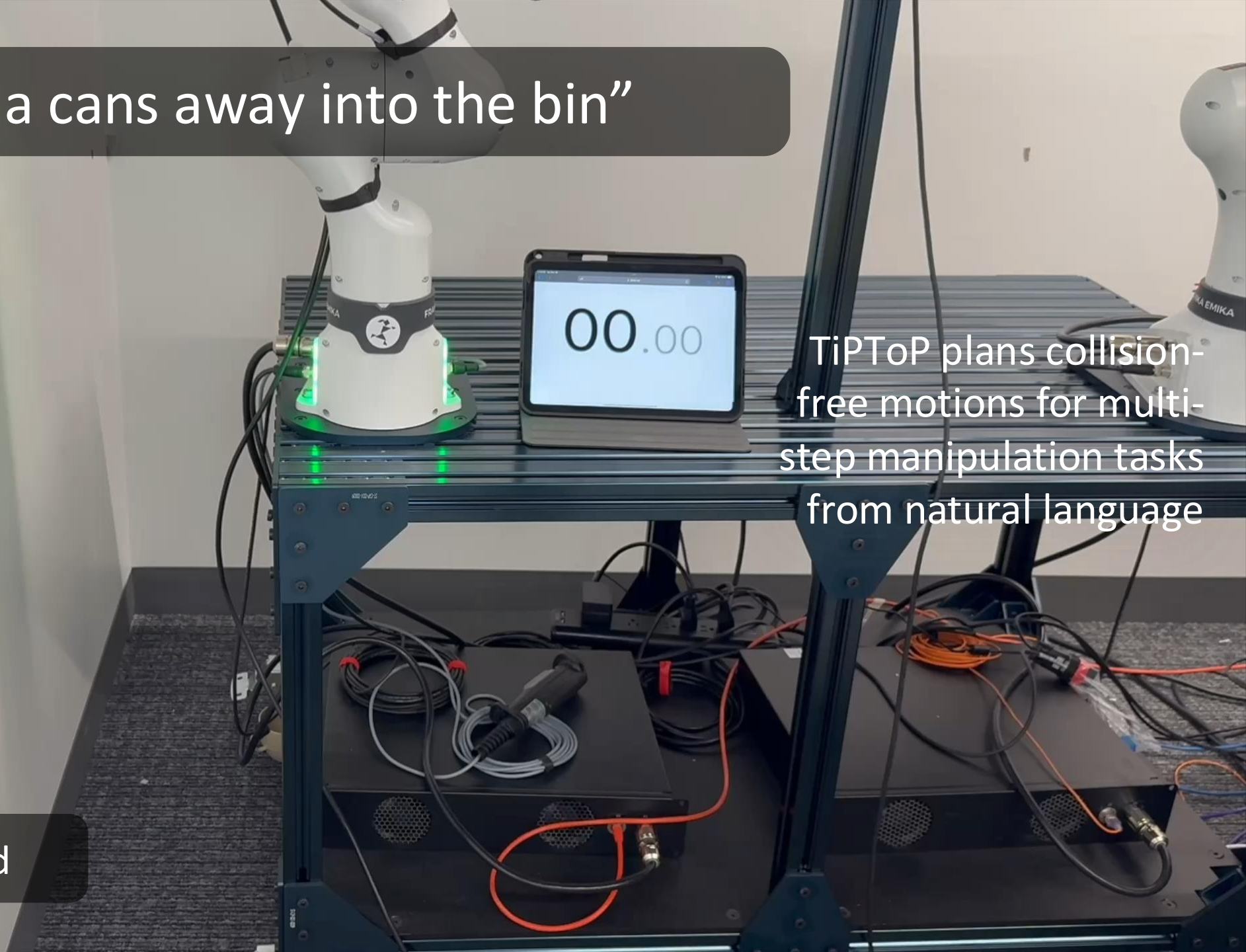
Slides adapted from Nishanth Kumar (thanks!)



“Throw the soda cans away into the bin”

TiPToP plans collision-free motions for multi-step manipulation tasks from natural language

autonomous, 2x speed



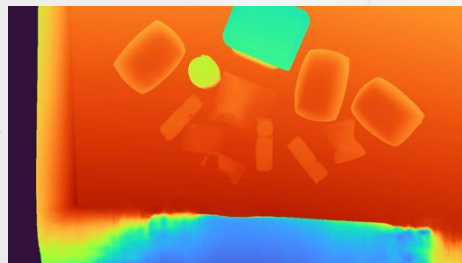
Image



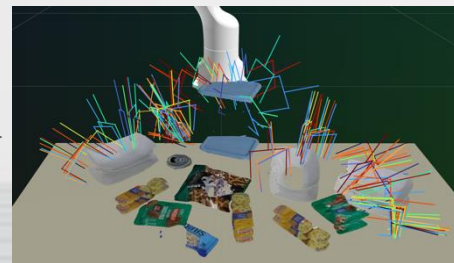
Task Instruction

“Serve peanut butter
crackers on each tray”

Depth Prediction



Grasp Prediction



3D Vision Branch

Object Detection



Object Segmentation



Semantic Branch

Task and
Motion
Planner
cuTAMP

Task Instruction

“Serve peanut butter
crackers on each tray”

Image

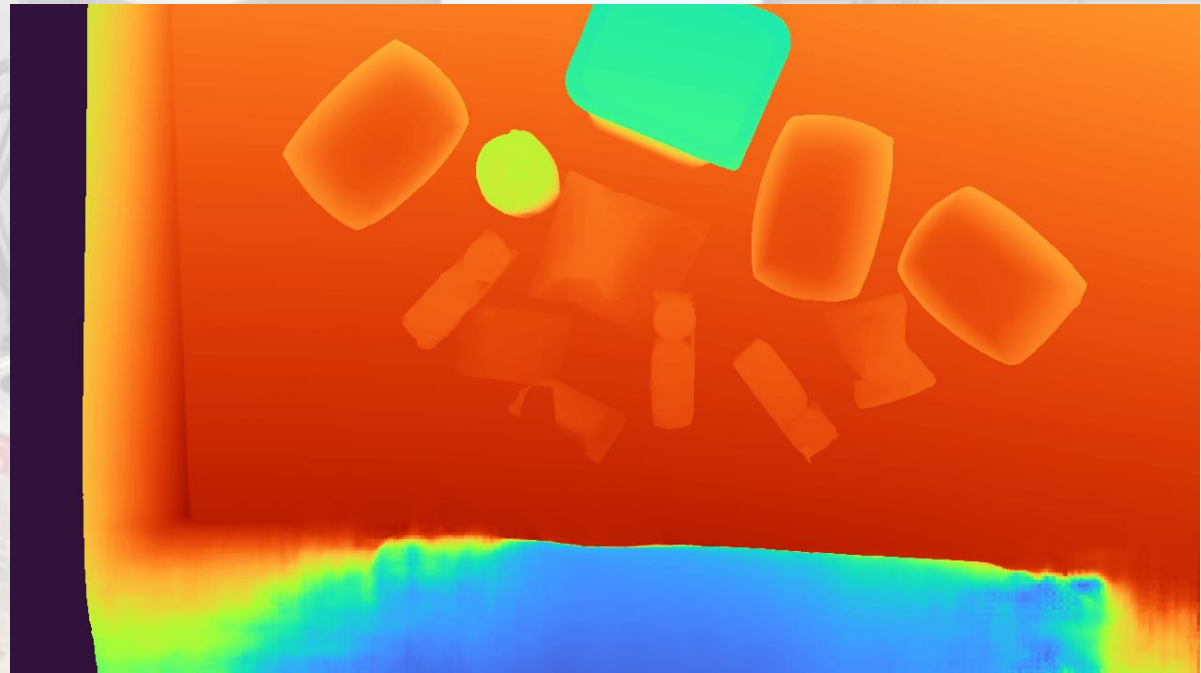


Wrist Camera

Image



Depth Prediction



Task Instruction

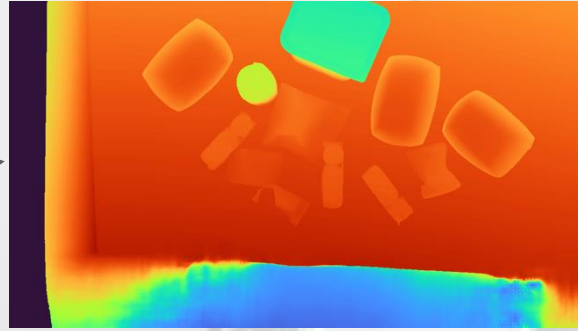
“Serve peanut butter
crackers on each tray”

“FoundationStereo”, Wen et. al. 2025.

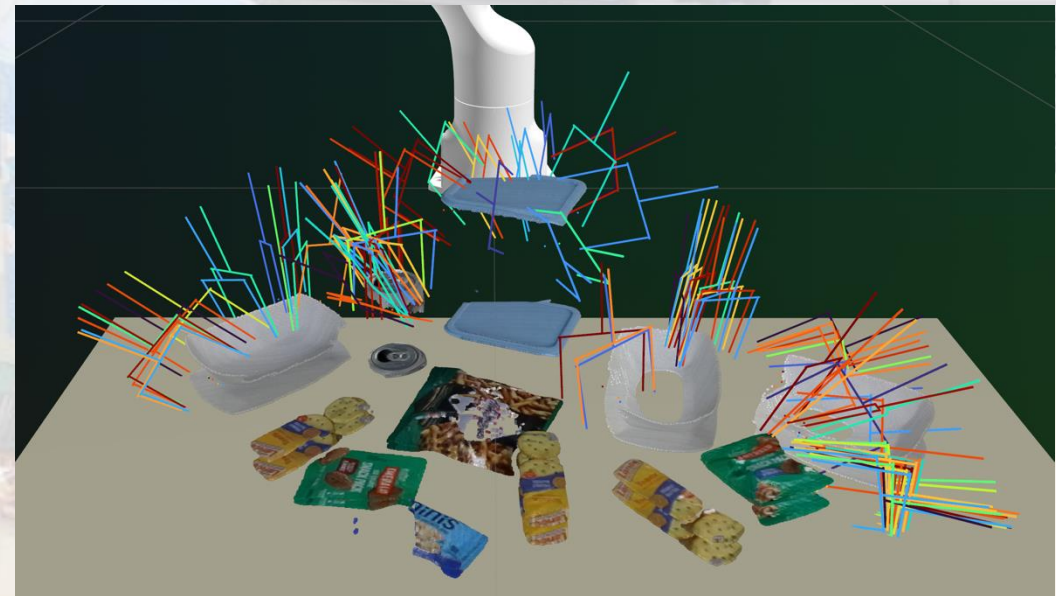
Image



Depth Prediction



Project to 3D
Point Cloud



Task Instruction

“Serve peanut butter
crackers on each tray”

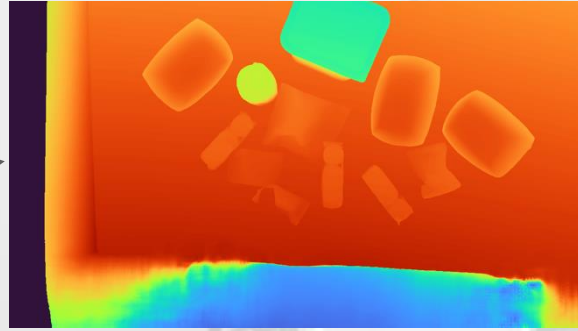
6-DOF Grasp Prediction

“M2T2”, Yuan et. al. 2023.

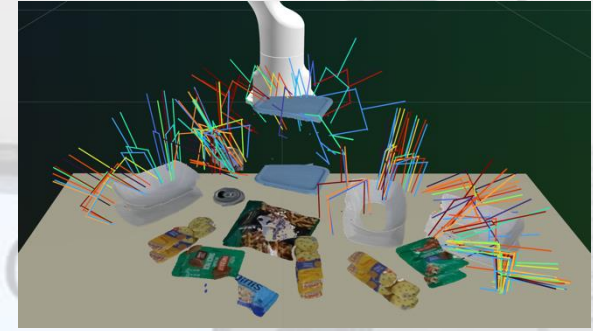
Image



Depth Prediction



Grasp Prediction



3D Vision Branch

Task Instruction

“Serve peanut butter
crackers on each tray”

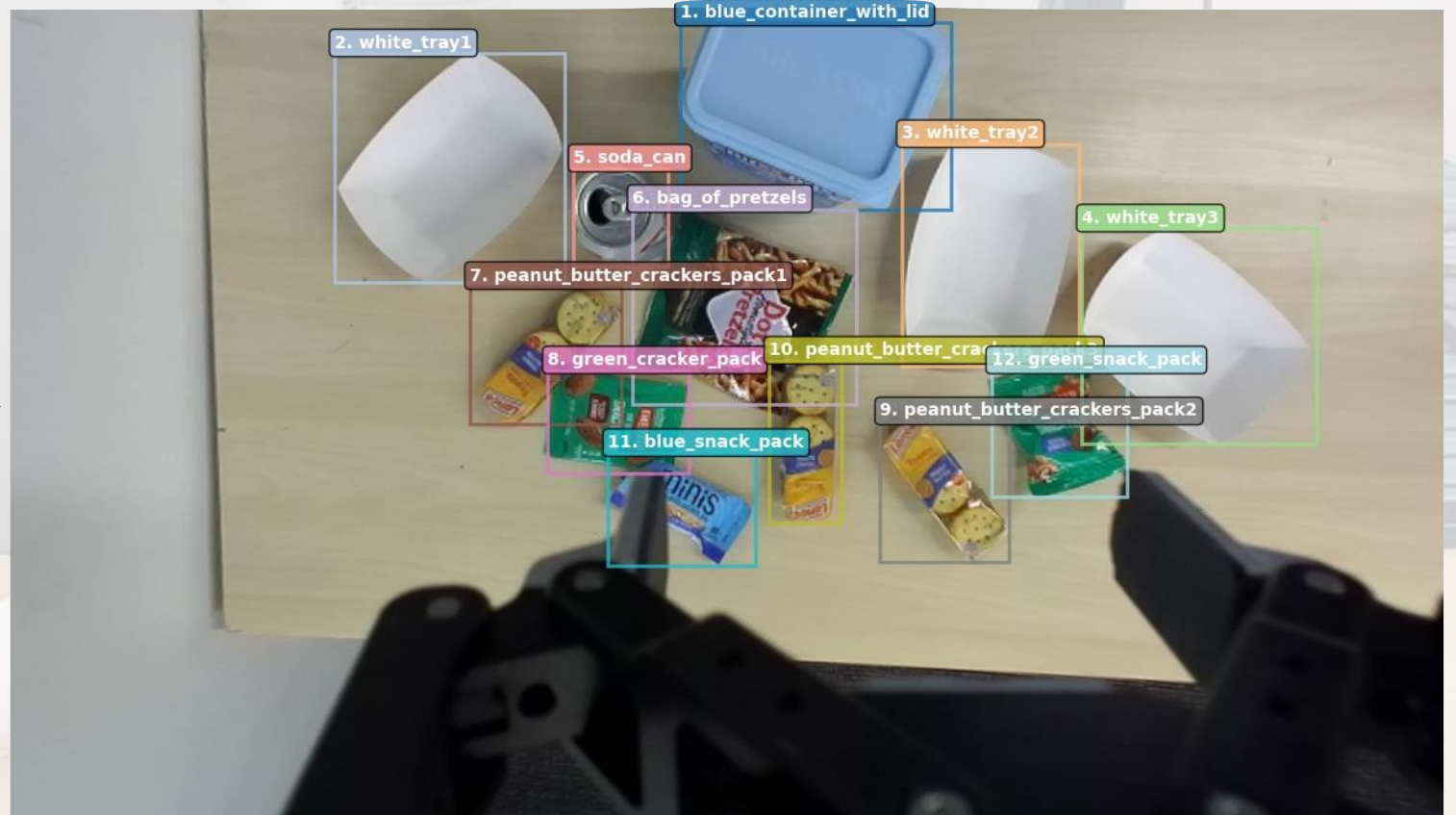


Image



Task Instruction

“Serve peanut butter
crackers on each tray”



Vision Language Model (VLM)-based
Object Detection

“Gemini 1.5 ER”, Google. 2025.

Object Detection

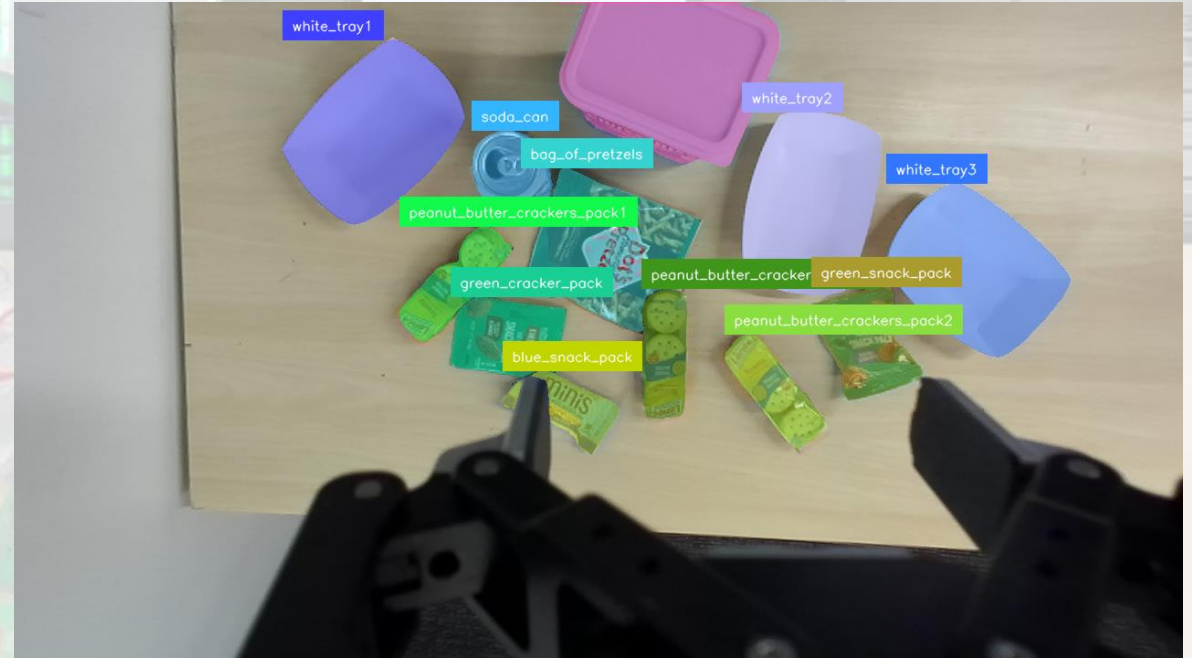
Image



Bounding
Boxes

Task Instruction

“Serve peanut butter
crackers on each tray”



Object Segmentation

“SAM 2”, Meta. 2024.

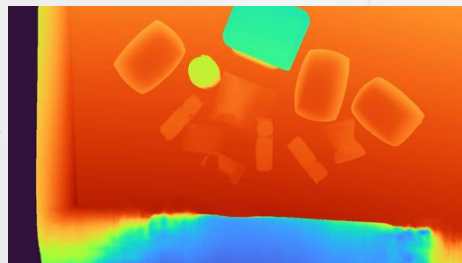
Image



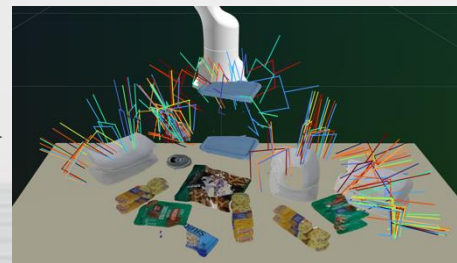
Task Instruction

“Serve peanut butter
crackers on each tray”

Depth Prediction



Grasp Prediction



3D Vision Branch

Object Detection



Object Segmentation



Semantic Branch

Task and
Motion
Planner
cuTAMP

cuTAMP Optimization

“Serve peanut butter crackers on each tray”



Initial Scene



*Slowed down for visualization

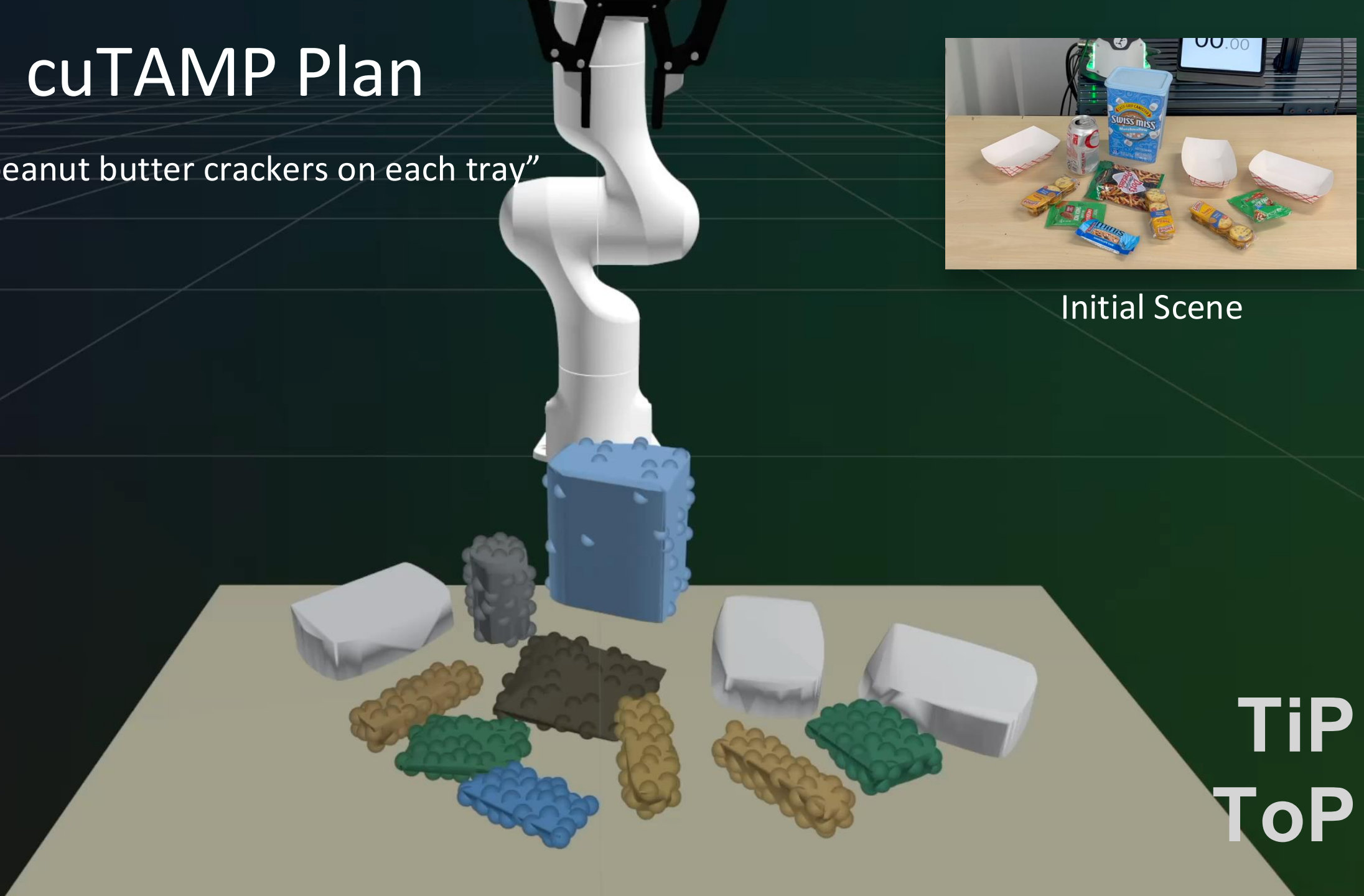
TiP
ToP

cuTAMP Plan

“Serve peanut butter crackers on each tray”



Initial Scene



TiP
ToP

Real-World Execution

“Serve peanut butter crackers on each tray”



Initial Scene

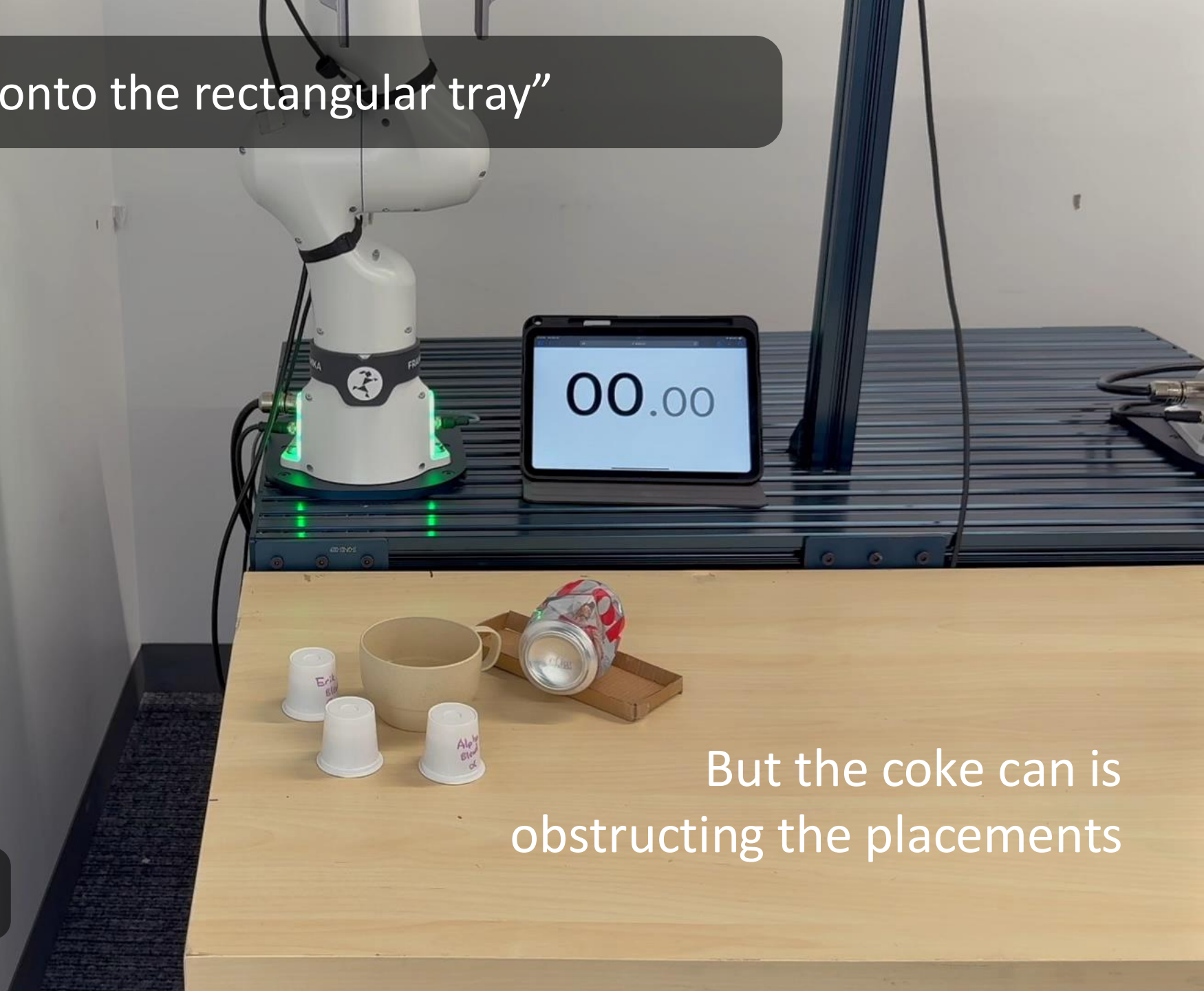
2x speed

TiP
ToP

“Pack the coffee pods onto the rectangular tray”

autonomous, 4x speed

But the coke can is obstructing the placements



TiPTOP

A pip-installable compositional manipulation system!

Generalizes to unseen objects, environments, and embodiments



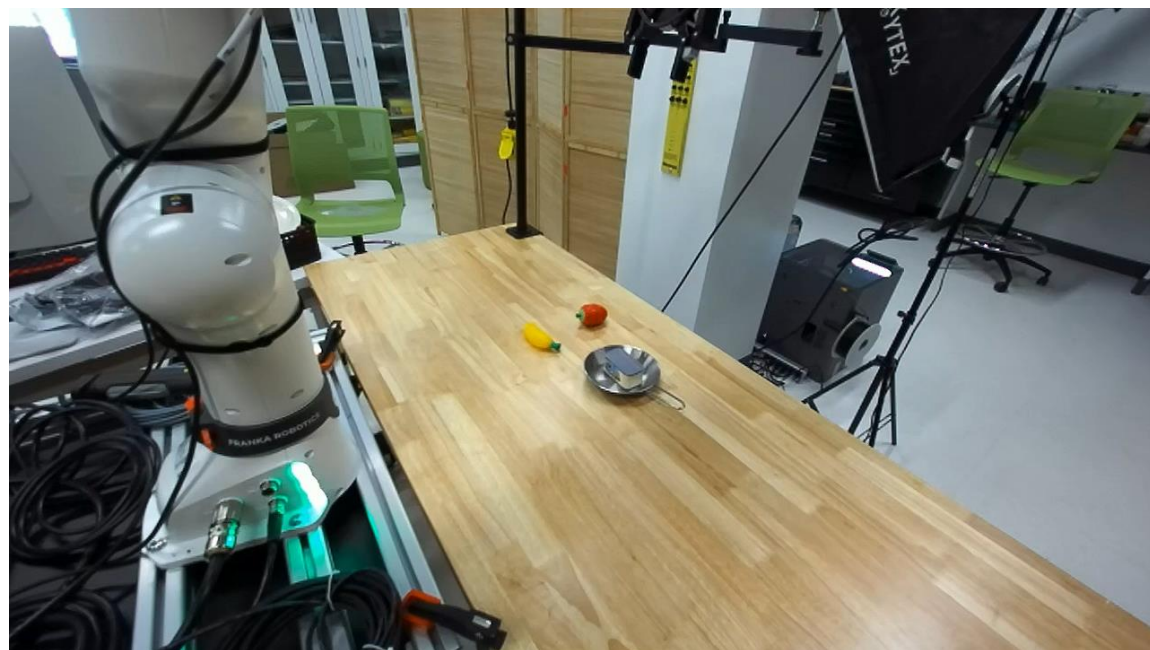
Running TipTop at Princeton

Pack only the fruits into the pan

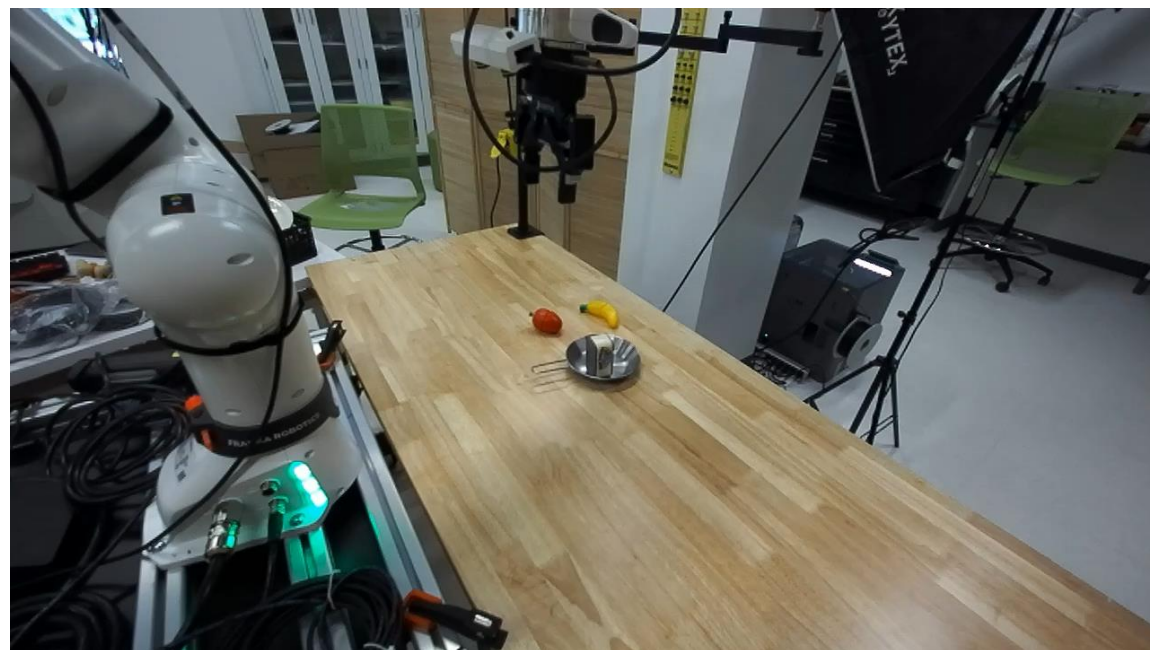


Samrat Sahoo

TipTop



Pi-0.5



Thanks! See Surveys For More

“Integrated Task and Motion Planning.” Caelan Reed Garrett, Rohan Chitnis, Rachel Holladay, Beomjoon Kim, Tom Silver, Leslie Pack Kaelbling, Tomás Lozano-Pérez. ARCRAS 2021.

“A Survey of Optimization-based Task and Motion Planning: From Classical To Learning Approaches.” Zhigen Zhao, Shuo Cheng, Yan Ding, Ziyi Zhou, Shiqi Zhang, Danfei Xu, Ye Zhao. IEEE/ASME Transactions on Mechatronics 2024.